Francesco AVANZI (*), Carlo DE MICHELE (*) & Antonio GHEZZI (*)

ON THE PERFORMANCES OF EMPIRICAL REGRESSIONS FOR THE ESTIMATION OF BULK SNOW DENSITY

ABSTRACT: AVANZI F., DE MICHELE C., & GHEZZI A., On the performances of empirical regressions for the estimation of bulk snow density ¹. (IT ISSN 0391 – 9838, 2015)

Snow covers are a seasonal reservoir of water in the solid form. The snowpack accumulates during winter and melts during spring and summer. This process rules streamflow timing and amount in many catchments in temperate areas. The amount of water mass in a snow cover is usually measured as Snow Water Equivalent (SWE, in kg m⁻² or mm w.e.), i.e. the mass of liquid water which would result from the complete melting of that snow cover. To calculate SWE, evaluations of snow depth and bulk snow density are needed. A widely applied solution in conditions of data scarcity implies the measurement of snow depth, and the prediction of bulk snow density using multiple empirical regressions involving, as predictors, a set of proxy variables, such as air temperature, wind velocity, elevation, snow depth, or the age of the snow cover. Here, we reviewed 18 regressions used in the Literature to estimate mean bulk snow density. We compared the estimates of these regressions versus continuous-time measurements of daily bulk snow density collected in western US by the SNOTEL network using snow pillows. This analysis shows that the average percentage difference between predictions and data is around 25% - 45%. In addition, this difference increases with elevation. This shows that particular care is due when using these approaches, especially at high elevations, where snow plays a relevant role in the local hydrologic regime.

 $\ensuremath{\mathsf{KEY}}$ Words: Snow density, SWE, Snow depth, empirical regressions, SNOTEL.

RIASSUNTO: AVANZI F., DE MICHELE C. & GHEZZI A., Sulle prestazioni di regressioni empiriche per la stima della densità della neve. (IT ISSN 0391 – 9838, 2015)

Le coltri nivali sono un serbatoio stagionale di acqua in forma solida. Queste coltri si accumulano durante l'inverno e fondono durante la primavera e l'estate. Tale processo governa la tempistica e l'ammontare della portata fluviale in molti bacini nelle aree temperate del nostro pianeta. La massa di una coltre nivale è solitamente misurata come equivalente idrico nivale (SWE, in kg m⁻² o mm w.e.). Questa quantità può essere definita come la massa d'acqua allo stato liquido che risulterebbe dalla fusione completa della coltre nivale. Per calcolare SWE, è necessario conoscere l'altezza di neve e la densità media del manto di neve. Una soluzione ampiamente applicata nel caso in cui i dati a disposizione siano scarsi implica la misura dell'altezza di neve e la stima della densità media a partire da regressioni multiple che considerino come predittori un insieme di variabili come la temperatura dell'aria, la velocità del vento, la quota, l'altezza di neve o l'età della neve rispetto alla prima deposizione. In questo contributo, selezioniamo 18 regressioni empiriche ricavate in letteratura per stimare la densità media di una coltre nivale. Compariamo quindi le stime di queste regressioni con misure tempo-continue di densità media raccolte negli Stati Uniti occidentali dalla rete SNOTEL mediante l'uso di snow pillows. L'analisi mostra che la differenza percentuale media tra le predizioni e i dati è circa il 25% – 45% della misura. Inoltre, questa differenza cresce con la quota. Questo mostra che è necessario prestare particolare attenzione all'uso di queste regressioni, specialmente ad alte quote, dove la neve gioca un ruolo particolarmente importante nel regime idrologico.

TERMINI CHIAVE: Densità della neve, SWE, Altezza della neve, Regressioni Empiriche, SNOTEL.

INTRODUCTION

Snow Water Equivalent is widely used to quantify the mass of a snow cover at a given location, or over a given area (DeWalle & Rango, 2011; Jonas & alii, 2009; Bavera a& De Michele, 2009; De Michele & alii, 2013; Bavera & alii, 2014). This evaluation is challenging, especially over a complex terrain, since accumulation and ablation dynamics of seasonal snow covers are highly variable in space and time (Grünewald & alii, 2010; Mott & alii, 2011; Scipiòn & alii, 2013; Mott & alii, 2014). Nonetheless, SWE plays a crucial role when assessing water availability in snow dominated catchments (Grünewald & alii,

^(*) Dipartimento di Ingegneria Civile e Ambientale, Politecnico di Milano, Milano, Italy, francesco.avanzi@polimi.it, ecohys.blogspot.it

^(*) Dipartimento di Ingegneria Civile e Ambientale, Politecnico di Milano, Milano, Italy, carlo.demichele@polimi.it

^(*) Dipartimento di Ingegneria Civile e Ambientale, Politecnico di Milano, Milano, Italy, antonio ghezzi@polimi.it

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2013). Therefore, describing its time and space dynamics represents one of the most important and addressed tasks in snow hydrology (DeWalle & Rango, 2011; Grünewald & *alii*, 2013).

SWE (in m w.e.) is defined as $\rho \cdot b/\rho_w$, where ρ is bulk snow density (usually, in kg m⁻³), b is snow depth (in m), and ρ_w is liquid water density (usually, in kg m⁻³). Estimations of this quantity can be obtained by means of direct measurements or models. Measuring techniques include, among others, 1) manual sampling, 2) automatic measurements, able to return continuous-time measurements of this quantity (Johnson, 2004; Egli & alii, 2009; Morin & alii, 2012; Johnson & alii, 2014), and 3) remote sensing techniques (Dong & alii, 2005; Dietz & alii, 2012). Models include detailed snowpack representations (e.g., CRO-CUS, Brun & alii (1989, 1992); Vionnet & alii (2012)), SNOWPACK (Bartelt & Lehning (2002); Wever & alii (2014)) or ALPINE3D (Lehning & alii (2006); Bavay & alii (2009, 2013)), one-layer schemes (e.g., Tarboton & Luce (1996): Ohara & Kayvas (2006): De Michele & alii (2013); Avanzi & alii (2014b)), or statistical descriptions, such as Lòpez Moreno & Nogùes-Bravo (2006); Bavera & De Michele (2009); Bavera & alii (2012, 2014).

Measurements of SWE can be either time consuming (as in the case of manual sampling, Jonas & alii (2009)), affected by sparse distribution (as in the case of point automatic techniques, Guan & alii (2013)), or at first very onerous, as in the case of remote sensed networks. On the contrary, the application of models, of different complexity, is often limited by data availability (McCreight & Small, 2014) and/or computational efforts (Bavera & alii, 2014). A widely applied solution in conditions of data scarcity implies the calculation of SWE basing on the measurement of snow depth and the estimation of bulk snow density by means of multiple regressions. These involve as predictors a set of proxy variables, such as snow depth, air temperature, elevation, aspect or the age of the snowpack (Meløysund & alii, 2007; De Michele & alii, 2013; McCreight & Small, 2014). These predictors are usually more diffuse than direct measurements of SWE.

This solution has been widely used in the Literature and is currently encouraged by the large diffusion of simple and cheap automatic measurements of snow depth (e.g., ultrasonic depth sensors, see Gubler (1981); Ryan & alii (2008); McCreight & Small (2014)). Moreover, bulk snow density dynamics at a site are often marked by a reduced year-to-year variability (Mizukami & Perica, 2008), which favors the applications of these regressions, when calibrated locally. Since most of existing regressions have been usually calibrated over a given area/location, they are also function of local climatic conditions and snow types (see Sturm & alii (1995) for a classification). As an example, it is likely that a regression that has been calibrated over alpine snow will return worst performances in areas of, e.g., maritime snow. It is also expected that elevation will play a role since, e.g., higher sites are usually characterized by colder climatic conditions, a longer accumulation season and, therefore, different snow conditions in time. However, systematic comparisons between

regression predictions and data have been limited in the past due to scarce availability of measurements.

Nowadays, automatic devices, such as snow pillows, are widely used all around the world to monitor *SWE*. They let to investigate bulk snow density dynamics at different time resolutions (Avanzi, 2011; McCreight & Small, 2014; Avanzi & *alii*, 2014b). An example is represented by the SNOTEL network (Serreze & *alii*, 1999), which carries out systematic measurements of snow depth, *SWE*, precipitation and air temperature in more than 800 sites within the western United States.

Here, we investigate the performances of empirical regressions against data of bulk snow density. We consider, at this scope, 18 regressions and multi-year data from 10 sites located in the western United States and belonging to the SNOTEL newtork. Sites are concentrated in an area including Idaho, Montana and Utah, and placed along an altitude gradient covering an elevation range between 1400 and 2700 m a.s.l.. This choice let on one hand to minimize effects due to latitude and longitude, while on the other hand it let to investigate how regression performances vary with site elevation.

The work is organized as follows: in Section 2, data are presented. In Section 3, we introduce the regressions considered and we discuss the evaluations of their performances against the data. In Section 4, the results of this analysis are reported and discussed.

DATA

The SNOTEL network (Serreze & alii, 1999; Mizukami & Perica, 2008; Avanzi & alii, 2014b) collects automatically weather and snow data, at daily and hourly resolutions, in approximately 800 sites within 13 States of western US. Data are freely available at http://www.wcc.nrcs.usda.gov/snow/ and have been widely used in the Literature to investigate, model and predict snowpack dynamics (Serreze & alii, 1999; Mote, 2003; Bales & alii, 2006; De Michele & alii, 2013; Avanzi & alii, 2014b).

At each site, *SWE* data are collected using a snow pillow, which measures the weight of the snow cover overlying a plate placed on the ground surface (Cox & *alii*, 1978; Johnson & Schaefer, 2002; Johnson, 2004; Johnson & Marks, 2004). Snow depth is measured, in proximity of the snow pillow, using an ultrasonic depth sensor, while precipitation is collected by means of a heated rain gauge. Air temperature is measured using a thermistor.

Here, we considered daily data from ten different sites placed in Idaho, Montana and Utah, and a time period of ten water years (2004-2013). We report in Table 1 the name and elevation of the different sites. The time period considered guarantees continuity in the time records of the data, i.e. *SWE*, snow depth and air temperature.

Daily data are subjected to a semi-automatic quality check operated by SNOTEL staff during and at the end of each Water Year (Avanzi & *alii*, 2014b). Therefore, no additional quality check has been performed.

TABLE 1 - The selected SNOTEL sites

ID	SNOTEL number	SNOTEL name	Elevation (m ASL)
S1	989	Moscow Mountain	1433
S2	319	Bear Basin	1631
S3	530	Hoodoo Basin	1845
S4	978	Bogus Basin	1933
S5	1013	Temple Fork	2258
S6	860	White Elephant	2350
S7	374	Bug Lake	2424
S8	576	Lehmi Ridge	2470
S9	450	Dollarhide Summit	2567
S10	318	Beagle Springs	2700

METHODS

REGRESSIONS CONSIDERED

We evaluate the performances of 18 empirical regressions in estimating bulk snow density during an entire season. Note that these regressions do not represent an exhaustive review of all the available formulations.

We collected regressions which consider simple predictors, i.e. variables which are either directly measured at a standard SNOTEL site (e.g. snow depth and air temperature), or derivable from other measurements (e.g., the age of snow). These include also topographic variables, such as site elevation, slope or aspect, which can be obtained either by contacting SNOTEL staff or from a cartographic resource. As a consequence, some relations (see e.g. Bilello (1969), Elder & alii (1998) or Meløysund & alii (2007)) have been neglected. These include, as predictors, some variables that are not collected in all the sites considered, like wind velocity, radiation or relative humidity.

We report in Table 2 the regressions considered in this analysis, and the Literature reference for each of them. In

Table 2 - The empirical regressions used in this study. b is snow depth in m, t is time in days, z is elevation in m, I is the local slope, t^* are days from 1st September, T is air temperature, A is site aspect, \overline{t} are days from 1st April and T_S is snow average temperature. a, b, C and B are parameters.

ID	Equation	Reference	
1	$\rho = a \cdot h + b$	Jonas & <i>alii</i> (2009)	
2	$ \rho = 148 + 105 \cdot h \text{ if } h \le 2 \text{ m} $ $ \rho = 358 \text{ if } h > 2 \text{ m} $	Lundberg & alii (2006)	
3	$\rho = 180 \cdot h + 90 \ if \ h < 0.6 \ \mathrm{m}$	Gavrilev (1965)	
4	$\rho = 158 \cdot \log(h) + 376$	Tabler (1980)	
5	$ \rho = 27 \cdot h + 358 \text{ if } h \le 1.7 \text{ m} $ $ \rho = 404 \text{ if } h > 1.7 \text{ m} $	Sand & Killingtveit (1983)	
6	$ \rho = 72 \cdot h + 275 \text{ if } h \le 1.7 \text{ m} $ $ \rho = 397 \text{ if } h > 1.7 \text{ m} $	Sand & Killingtveit (1983)	
7	$ \rho = 70 \cdot h + 286 \text{ if } h \le 2.25 \text{ m} $ $ \rho = 443 \text{ if } h > 2.25 \text{ m} $	Marchand (2003)	
8	$\rho = 522 - \frac{204.7}{h} \cdot \left(1 - e^{-h/0.673}\right)$	Tabler & <i>alii</i> (1990)	
9	$\rho = 488 - \frac{204.7}{h} \cdot \left(1 - e^{-h/0.673}\right)$	Pomeroy & Gray (1995)	
10	$\rho = 2.5692 \cdot h + 331.81$ $\rho = 390 \text{ if } 2.5692 \cdot h + 331.81 > 390$	Marchand & Killingtveit (2004)	
11	$ \rho = a + b \cdot t $	Mizukami & Perica (2008)	
12	$\rho = C \cdot \sqrt{h} + B$	Meløysund & alii (2007)	
13	$\rho = 352 + 75 \cdot \ln(h)$	Gustafsson & alii (2012)	
14	$\rho = 450 - \frac{204.7}{h} \cdot \left[1 - e^{-h/0.673}\right]$	Pomeroy & <i>alii</i> (1998)	
15	$\rho = 1.03 \cdot \overline{t} + 316$	Elder & <i>alii</i> (1991)	
16	$\rho = 0.038 \cdot z + 0.649 \cdot t^* - 1.434 \cdot I + 145.03$	Bavera & De Michele (2009)	
17	$\rho = (90 + 130\sqrt{h}) \cdot (1.5 + 0.17 \cdot \sqrt[3]{T})$	Meløysund & <i>alii</i> (2007)	
18	$\rho = 134 + 0.54 \cdot t^* + 0.06 \cdot z + 0.1 \cdot A - 1.45 \cdot I + 6.09 \cdot T_S$	Martinelli & <i>alii</i> (2004)	

the same table, symbols are also explained. Regression 1 correlates snow depth with bulk snow density, using two parameters, a and b, which vary according to site elevation and the period of the year, at monthly scale. Regressions from 2 to 9, reported in the review by Lundberg & alii (2006), refer to snowpack "before any larger melt has taken place" (Lundberg & alii, 2006). However, in this application, we have applied these relations over the entire period to evaluate how an improper use of this typology of relations may affect the predictions. This is because inferring the exact timing of melting is a recurring problem in practical applications. Marchand & Killingtveit (2004) (regression 10) suggests an alternative formulation in case of forest sites, also. Nonetheless, we considered here the "open-field" option since it looks more appropriate for SNOTEL sites. Coefficients of regression 11 change with the location (western US region) and the season (i.e., midwinter or spring). Regression 17 is a simplification of a more general equation that considers also wind velocity.

Regression 18 needs an evaluation of snow average temperature T_S . Here, we considered an approximate approach that is aimed at replicating the expected dynamics of seasonal snow temperature. In particular, we assumed the snowpack as isothermal at 0°C when air temperature is positive since in this condition it is likely that snow is melting, while, when air temperature is negative, we impose a linear behavior of air temperature between snow surface, where snow temperature is imposed equal to air temperature T, and soil surface, where snow temperature is imposed equal to 0°C, as it has been largely observed in alpine areas, see e.g. Filippa & alii (2014). Thus, if measured air temperature is positive, $T_S = 0$ °C, while if air temperature is negative, $T_s = T/2$. Note that this represents only a very simplified approach, used in conditions of data scarcity, since many processes involved in snow energy balance are not considered. In the same regression, topographic predictors, such as slope angle or aspect, have been estimated from site characteristics.

EVALUATION OF THE PERFORMANCES

The evaluation of the performances of the regressions has been operated as follows: for each site, and each water year, we calculated the predicted bulk snow density according to all the equations given in Table 2, at daily scale.

Then, we considered the daily measurements of ρ , and we evaluated the percentage difference between data and each regression. We define this percentage difference as

 $\Delta = |\frac{\rho_{mod} - \rho_{obs}}{\rho_{obs}}|.$ In the following, we will discuss how this

difference varies with elevation, considering, for each site (i.e., each elevation) the average value of Δ ($\Delta_{\rm med}$) calculated merging all available years and all the different regressions. In addition, we also evaluated possible $\Delta_{\rm med}$ trends with elevation considering the distinction between mid-winter and spring estimates. The periods corresponding to mid-winter and spring were defined at each site by evaluating the average date of maximum accumulation. Mid-winter is therefore the period before the peak accu-

mulation, while spring is the period after this date. Note that this date is assumed constant at each site during the entire period of study.

RESULTS AND DISCUSSION

THE COMPARISON BETWEEN DATA AND MODELS

We report in figure 1 three examples of observed (black dots) and estimated (gray lines) bulk snow density, for three sites among the ten considered, and the same water year (2013). In particular, in figure 1(a) the comparison is made at the lowest site, S1 (i.e. Moscow Mountain, 1433 m a.s.l.), while in figure 1(b) we report the same comparison for a medium elevation site, i.e. S5 (Temple 185 Fork, 2258 m a.s.l.). In figure 1(c), the comparison is reported for the highest site, S10 (i.e., Beagle Springs, 2700 m a.s.l.).

figure 1 shows that data and estimates are systematically of the same order of magnitude. Nonetheless, while at S1 (fig. 1a) the range of variation of the estimates matches the

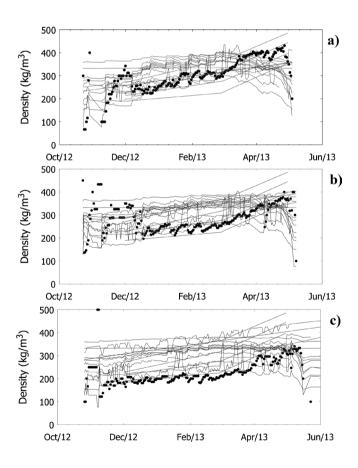


Fig. 1 - Examples of observed (black dots) and predicted (gray lines) bulk snow density, or three sites among the ten considered, and the same year (2013). In particular, panel 1(a) reports the comparison at S1 (i.e. Moscow Mountain, 1433 m a.s.l.), panel 1(b) the comparison is reported at S5 (i.e. Temple Fork, 2258 m a.s.l.), while in panel 1(c) the comparison is made at S10, Beagle Springs (2700 m a.s.l.).

range of variation of data, at S10 (fig. 1c) a clear, and systematic, overestimation of snow density is visible. In particular, observed snow density acts as the inferior boundary of estimates during the accumulation season. S5 shows an intermediate behavior between S1 and S10.

Most of the regressions considered here assume a simple dependency between bulk snow density and snow depth. As a consequence, bulk snow density increases with snow depth. Such a relation is justified during the accumulation season, when overburden pressure of snow causes an increasing of snow density because of settling (Henry, 1917; Kojima, 1967; Mellor, 1977). Nonetheless, the same simple relation entails that bulk snow density decreases during spring time, since snow depth decreases due to the melting of the porous structure. On the contrary, observed bulk snow density increases monotonically during both the seasons, due to the overlap between mechanical deformations, metamorphisms and pore saturation by liquid water (Cresseri & alii, 2010; Avanzi, 2011). It follows that, generally, observed and estimated densities dynamics in time agree during mid-winter (i.e., "before any larger melt has taken place" (Lundberg & alii, 2006)), but disagree during spring. This is well known in the Literature (Lundberg & alii, 2006; Jonas & alii, 2009; McCreight & Small, 2014) and explains why users must pay attention to the validity period of any regression used. On the contrary, relations such as those proposed by Mizukami & Perica (2008) or Bavera & De Michele (2009), which consider time as the main predictor, do not show such an issue.

As a clear example of this behavior, we report in figure 2 vearly trajectories of average daily vertical stress in the snow-

0.1 σ (bar, log scale)

Fig. 2 - Examples of relations between average vertical stress σ and simultaneous bulk snow density ρ at site S6, evaluated on a daily base. Different symbols stand for different water years of data. The two black lines indicate the two boundaries of the σ – ρ zone reported by Mellor (1974) for natural densification of snow deposits of for temperatures between -1 °C and -48°C.

400

Bulk snow density (kg/m³)

600

0.001

100

pack, calculated as $\sigma = \frac{\rho_W gSWE}{2}$, where g is the gravitational acceleration (Avanzi, 2011), as a function of simultaneous bulk snow density, at a median elevation site, S6, during the water years 2004 - 2008. σ (approximated imposing a linear profile of stresses in the snowpack) increases during the accumulation period, because of new events, hence increasing snow depth, but decreases during the melting period, because of ablation. On the contrary, ρ increases monotonically during the season. For low densities (say, ≤ 300 kg m⁻³, typical of the accumulation period, Mizukami & Perica (2008)), trajectories are almost coincident, and rather coherent with the $\sigma - \rho$ area reported by Mellor (1974) for natural densification of snow deposits for temperatures between -1°C and -48C (black lines). On the contrary, during the ablation period (i.e., for higher densities), the trajectories never retrace the same path, exiting Mellor's zone and, generally, showing much more scatter. This irreversible behavior, marked by hysteresis, is probably one of the greatest limitation of snow depth-based regressions. A wider discussion about this topic can be found, for example, in Jonas & alii (2009), Avanzi (2011) and Avanzi & alii (2012).

ELEVATION-PERFORMANCES RELATION

In figure 3, we report Δ_{med} as a function of site elevation. This analysis shows that average percentage differences between data and regressions span between 25 % and 45 %. Such a difference is rather high, if we consider that seasonal bulk snow density is usually $\ge 70 - 100 \text{ kg} / \text{m}^3$ and $\le 500 \text{ kg}$

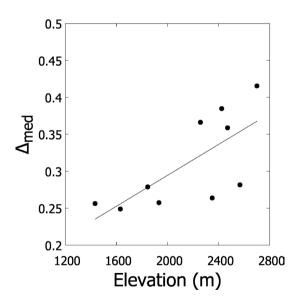


Fig. 3 - Average percentage difference between empirical regressions predictions and data as a function of site elevation.

/ m³, and that the year-to-year variability of this quantity is usually reduced (Mizukami & Perica, 2008). It is challenging to define a threshold for acceptable performances, since similar inter-comparison tests have been rarely performed in the past. In this contribution, we therefore provide an evidence that, when applying these relations with no reference to their area of calibration and/or period of validity, a Δ_{med} \geq 25% may be expected. In addition, Δ_{med} increases with elevation. A simple linear regression reads $\Delta_{\text{med}} = 0.0001046$ \cdot z + 0.08531 (R^2 = 0.5). Such a result, already discussed in figure 1, is particularly interesting since, usually, the relevance of snow precipitation, thus SWE, increases with elevation (Avanzi & alii, 2014a). Thus, the uncertainty in SWE estimations by using simple regressions is maximum, where SWE plays a key role on local hydrology. The same analysis has been done in figure 4, by separating mid-winter (fig.

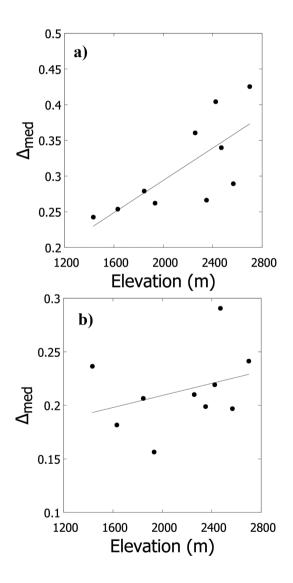


Fig. 4 - Average percentage difference between empirical regressions predictions and data during mid-winter (panel 4a) and during spring (panel 4b) as a function of site elevation.

4a) and spring (fig. 4b) data. The general pattern observed in figure 3 is confirmed when considering mid-winter data. The regression line reads $\Delta_{med}=0.0001131\cdot z$ - 0.06779 (R² = 0.53). On the contrary, spring-time data show a great scatter. In this case, the regression reads $\Delta_{med}=0.00002818\cdot z+0.1529$ (R² = 0.1). Surprisingly, Δ_{med} in spring time is reduced, if compared with mid-winter data. This is mainly due to the fact that, at all elevations, data and predictions are negatively correlated during spring. Therefore, local overestimations during the accumulation season are compensated during spring time, hence a reduced Δ .

CONCLUSIONS

We investigated the performances of 18 empirical regressions available in the Literature to estimate seasonal bulk snow density against measured data. We considered ten SNOTEL sites in western US, distributed along an altitude gradient, and ten water years of data (2004–2013). This analysis shows that:

- Observations and estimates are of the same order of magnitude, although a clear, and systematic, overestimation of snow density is visible at high elevations;
- Observed and estimated density dynamics in time agree during mid-winter, but disagree during spring, since most of the regressions consider snow depth as the unique predictor of snow density, and cannot reproduce the observed hysteresis in the snow depth – SWE relation (Jonas & alii, 2009);
- Average and maximum percentage differences between models and data are rather high, and increase with elevation.

This analysis shows that care is due when applying these relations, especially at high elevations.

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