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NUMERICAL FRAMEWORK FOR GEOMORPHOLOGICAL EXPERIMENTS

ABSTRACT: CAPOLONGO D., GIACHETTA E. & REFICE A., *Numerical framework for geomorphological experiments.* (IT ISSN 0391-9838, 2011).

In this work we present a numerical framework for simulation of surface processes and landforms. The model is called SIGNUM (Simple Integrated Geomorphological NUmerical Model) and is a Matlab, TIN-based landscape evolution model.

We use the model to show a few examples of simulated topographic surfaces evolved through application of mathematical expressions for hillslope and fluvial erosion, channel sediment transport and surface uplift.

A particular example is shown to reproduce a topographic feature similar to real landscapes, namely the approximately regular spacing of valleys at linear mountain fronts.

Although work in the field of computer simulation of geomorphological processes of landscape evolution is at its beginning, results and insights from models such as the ones we present are gaining more and more attention in the scientific community, justifying and encouraging increasing research efforts.

KEY WORDS: Landscape evolution, Numerical model, Thrust-faults, Valley spacing.

INTRODUCTION

Geomorphology is undergoing a period of great development, driven mainly by the availability of high-resolution topographic data, geochronological measurements and high-performance computing environments (Summerfield, 2005). Nowadays, geomorphologist are able, in an unprecedented manner, to analyze the Earth surface at the scale of the geomorphological processes acting on hillslopes and in fluvial systems (from 100s of m down to 1 m or less), to measure rates of landform evolution and to explore how physics and chemistry of surface processes influence different paths of landscape development. In addition, new methods in geochronology, such as cosmogenic nuclide and carbon-14 analysis, or optical stimulated luminescence, allow geomorphologists to date deposits, landforms, timing of events and rates of erosion and tectonic deformation with a time resolution not available before. Ultimately, these data can be used to derive and test theories of landscape evolution and to explore in a quantitative manner complex process dynamics, as for example how climate influences erosion, transport and deposition (Bull, 1991; Molnar, 2001), or the complex interplay between climate, tectonics, and erosion in mountain belts (e.g. Willet & *alii*, 2006; Boenzi & *alii*, 2008; Schiattarella & *alii*, 2008).

The necessity of linking surface processes and the resulting landforms in a quantitative manner, in complex and non linear geomorphological systems, is important also for prediction of future landscape evolution in term of geomorphological hazards and planning (Anderson 2008; Tucker, 2009).

This issue has driven researchers to develop several analytical and numerical models of landscape evolution in the past ten years. Analytical models have given important insights on how geomorphological systems operate, yet they suffer the distortion due to the reduced spatial (laboratory scale landforms) and temporal scale (few days or weeks of simulation correspond to thousands of years in real landscapes, Schumm & *alii*, 1987; Bonnet & Crave, 2006).

Since at least the 1980s, the term landscape evolution model (LEM) has been used to mean a description of the evolution of landscape over geologic times (Pazzaglia, 2003; Tucker, 2009). In the last two decades of the 20th century, several numerical models have been developed to describe the evolution of the Earth surface over time at different spatial and time scales. Recently, scientists coming from different disciplines (geomorphologists, hydrologists, biologists, physicists, etc.), are collaborating to wide joint international research efforts in numerical modeling such as the Community Surface Dynamic Modeling Systems (CSDMS, http://csdms.colorado.edu), to understand how topography, hydrology, vegetation and climate coevolve, through investigation of the combined effects and

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feedback of feature such as rainfall patterns or orography-controlled precipitation.

Landscape evolution models (LEMs) have mainly two purposes: 1) testing quantitative assumptions on processes acting on complex systems, thus providing a linkage between field measurable processes and their geomorphological implications on long term evolution, and 2) providing a support tool in geo-environmental studies and hazard prediction.

A review of landscape evolution numerical modeling is beyond the scope of this paper. Excellent reviews (Tucker, 2009 and reference there in) can be found in the recent literature.

Aims of this paper are: a) to outline the basic concepts behind state-of-the-art numerical modeling in geomorphology, and b) to present SIGNUM, our numerical framework for geomorphological experiment.

BASICS OF NUMERICAL LANDSCAPE EVOLUTION

The basics for numerical modeling in geomorphology are related to the principle of mass conservation, or continuity equation (Anderson, 2008; Tucker & *alii*, 2001), which can be written as:

$$\frac{dz}{dt} = -\nabla q_s + U(x, y, t), \tag{1}$$

where the left term is the time (t) rate of change of elevation (z) of a generic point of the surface, U is the tectonic uplift and q_s is the sediment flux per unit width (∇ is the divergence operator). The first term on the right includes several processes, describing different forms of erosion. The above mathematical relationship says in practice that in a control volume the rate of change of elevation is equal to the difference between the mass flux which comes in and the one which comes out. It is interesting to note that the form of equation (1) is not dependent on the time scale, which means that it can be applied to long-term landscape evolution as well as to short time scales. To solve the mass conservation equation (1), we need erosion and transport laws in numerical form for different geomorphological processes, and we also need representations in the same form of surface larger scale deformations, such as tectonic uplift, subsidence, or sea level change. The numerical representation of geomorphological processes are defined as Geomorphic Transport Laws (GTLs, Dietrich & *alii*, 2003). A GTL describe the average mass flux on a defined spatial and temporal scale at a point on the landscape due to a particular geomorphic process. In general, it has a physically based mathematical expression whose parameters can be possibly estimated in the field. A GTL describes how the average mass flux depends on topography, material properties, and other environmental factors (e.g. climate). When solved together with the continuity equation (1) given initial and boundary conditions (i.e. initial topography and constraints to the evolution imposed by the finite size of the model, respectively) and a function $f(x,y,z,t)$ that describes the surface uplift, it provides a physical basis for predicting how the topography will evolve. The solution of this equation is not straightforward and often needs numerical approaches. Among the most common GTLs are the ones used to represent fluvial and hillslope processes, which represent the first-order processes controlling landscape dissection. Dietrich and Perron (2006) provide a summary (table 1, p. 413) of the most applied GTLs in landscape evolution numerical modeling. Some widely used expressions for river incision into bedrock and slope dependent movements on hillslopes (linear and nonlinear diffusion) are reported in table 1. Those GTLs has been included as process modules in SIGNUM and have been used, coupled with simulated surface uplift due to thrusting, to solve the continuity equation in the experiments presented in sect. 4.1. Apart from these simple cases, at present there still is a relative lack of physically-based, field testable quantitative transport laws for many geomorphological processes. Work in this field of geomorphology is at its very beginning, and strongly motivates new research, new field studies and novel geomorphological investigations on real and simulated landscapes.

SIGNUM. A Matlab, TIN-based model

SIGNUM (Simple Integrated Geomorphological Numerical Model) is a TIN-based landscape evolution model: it is capable of simulating sediment transport and erosion by river flow at different space and time scales. It is a multi-process numerical model written in the Matlab high level programming environment, providing a simple and integrated numerical framework for the simulation of some important processes that shape real landscapes. Particularly, at the present development stage, SIGNUM is capable of simulating geomorphological processes such as hillslope diffusion, fluvial incision, tectonic uplift or changes in base-level and climate effects in terms of precipitation. We

TABLE 1 - Numerical expressions for the main surface processes simulated in SIGNUM

PROCESS	EQUATION	PARAMETERS	REFERENCES
Non linear diffusion	$\left.\frac{\partial z}{\partial t}\right\|_{diffusion} = \frac{K_{nl}\nabla z}{1-\left(\frac{\|\nabla z\|}{S_c}\right)^2}$	• K_{nl} = non-linear diffusion constant (m^2/y) • S_c = critical slope (tgα)	(*Roering*, 1999)
Linear diffusion	$\left.\frac{\partial z}{\partial t}\right\|_{diffusion} = k_l \nabla^2 z$	K_l = linear diffusion constant (m^2/y)	(*Braun & Sambridge*, 1997)
Fluvial incision	$\frac{\partial z}{\partial t} = -K_i A^m S^n$	• K_i = bedrock incision constant (m/y) • A = upslope drainage area (m^2) • S = slope (tgα) • m & n = exponents (dimensionless)	(*Howard* ,1980) (*Stock & Montgomery*, 1999)
Tectonic uplift or Base level changes	$\frac{\partial z}{\partial t} = U(x,y,t)$	U = uplift rate (mm/y)	(*Tucker & alii*, 2001)

provide hereafter a brief description of SIGNUM basic concepts and operation. A fuller technical description is reported in another paper in preparation, while in the present work we focus on some applications of the model to state-of-the-art problems in landscape process research. Most landscape evolution models are based on grid-like representations of topographic surface (e.g. Vivoni & *alii*, 2005). This kind of terrain representation in terms of equal-sized square cells has the advantage of lending itself to finite difference solutions, but suffers of some drawbacks. Main restrictions are:

a) the features of relief must be represented with a constant spatial resolution, which often means the highest resolution for the feature or processes considered;
b) drainage patterns are restricted to either one or a combination of several among eight possible directions, separated by 45°. These choices are referred to in the literature as D8 or D-infinity drainage direction methods, respectively (e.g. Fairfield and Leymarie, 1991, Tarboton, 1997); in some cases this can produce an asymmetry leading to artifacts in drainage patterns (Braun & Sambridge, 1997);
c) grids hinder simulations of some geomorphological processes involving a significant horizontal component, such as river meandering or fault displacements.

TIN-based landscape evolution models, on the contrary, allow a variable spatial resolution and a dynamic discretization of the topographic surface, althought TINs are still interpolated surfaces. The conceptual framework of SIGNUM is analogous to many landscape evolution models recently described in the literature. Use is made of Matlab data structures and index arrays. These are used as outlined in papers such as Tucker & *alii* (2001). Two main structure concepts are used in SIGNUM, one related to the points on which the landscape height is sampled, the other connected to the TIN structure, usually a Delaunay triangulation, which is passed to the program at the beginning and is the basis for the simulated processes. Associated to these main structures are other support arrays which help in bookkeeping the links and indices. As in many LEMs, in our model all water and sediments leaving a cell are routed downstream in the direction of the steepest descent cell connected to it (fig. 1). The simulation is run on several user-defined cycles (epochs). At each new epoch the height of the TIN nodes is updated based on the processes selected to solve the mass conservation equation. At each cycle, terrain parameters (slope, contributing area) are recalculated (fig. 2).

MOUNTAIN FRONT VALLEY SPACING

We describe here a set of experiments of landscape evolution performed with SIGNUM, in which the use of a restricted set of surface evolution laws is able to produce certain statistical organization and order characteristics in the simulated landscape which resemble similar features observed on real landscapes.

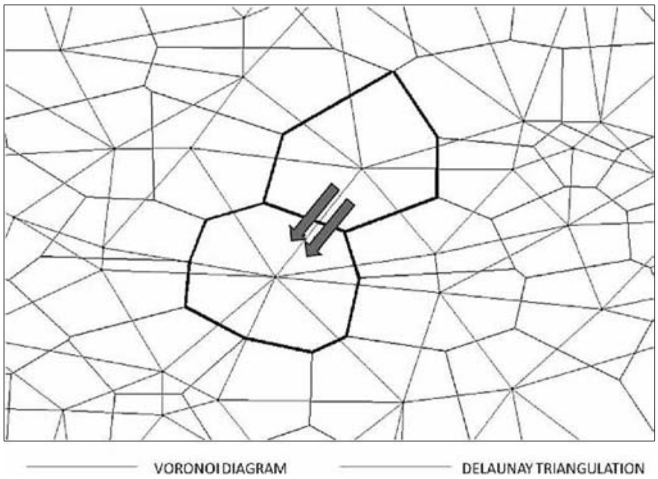

FIG. 1 - Schematic representation of water and sediment routing along TIN edges (light gray) and Voronoi diagram sides (dark gray) in SIGNUM.

The organization we explore here is a fascinating and yet not really well understood example of apparent regular spacing between transverse rivers in linear sections of mountain ranges Hovius (1996) analyzed the drainage of 11 different linear mountain belts worldwide. His study revealed that the outlets of the major transverse rivers at the front of these orogens are regularly spaced and also that their spacing S is on average proportional to the width W of the range (measured from the drainage divide to the front) following the relation $S = W/R$ (fig. 3). He also showed that the spacing ratio R of these ranges is distributed in a narrow range of values (1.91-2.23), around a mean of 2.1, despite strong differences in climate and rock uplift rates among the study test sites. Talling & *alii* (1997) found a higher mean value for R (2.5), but their study additionally incorporated basins that extend across 70% or more of the range width from drainage divide to mountain front, and included a much greater diversity of mountain topography.

The dynamic nature of the morpho-tectonic setting of mountain belts makes the identification of the mechanisms controlling the regularity of valley spacing problematic because the boundaries represented by the position of the range front and the range divide are not fixed, but will change with mountain belt widening. Moreover, the long time scale over which the drainage network organizes and the strongly dynamic environment with high denudation rates typical of mountain belts prevent the preservation of deposits and morphological features. This makes and ideal case to be explored by numerical modeling.

The question of what causes the regularity of river spacing in linear sections of mountain belts around the world has been explored by a few studies (e.g. Hovius, 1996; Talling & *alii*, 1997; Castelltort & Simpson, 2006; Walcott & Summerfield, 2009). One proposed explanation is that the spacing ratio is maintained as range width increases through drainage reorganization (e.g. Bonnet,

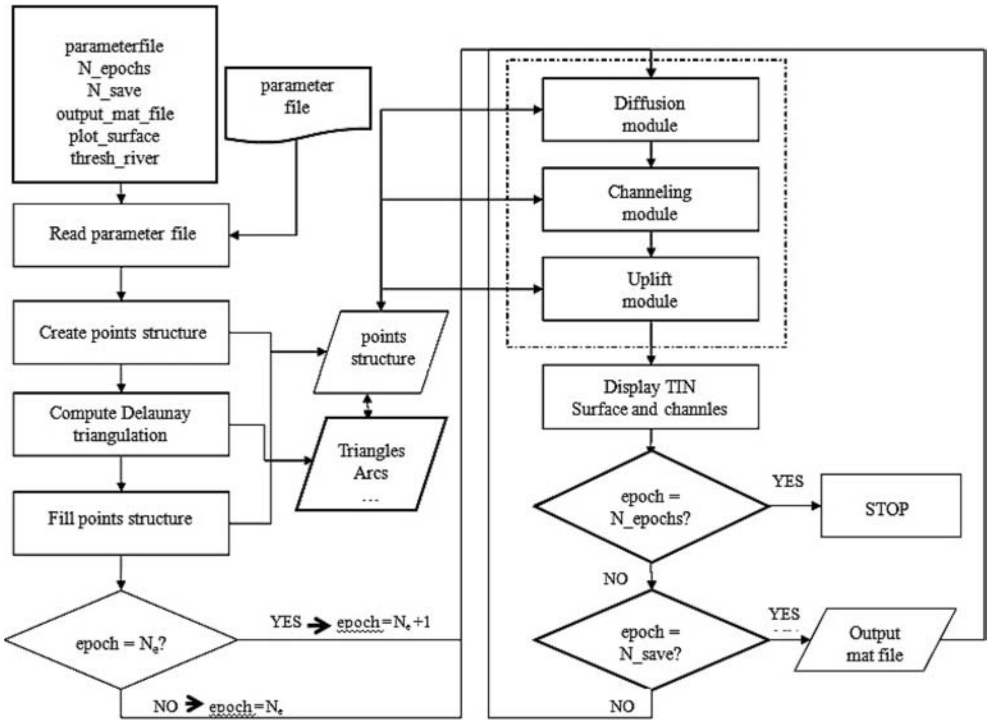

FIG. 2 - Flow chart showing the SIGNUM processing structure.

2009). For the spacing ratio to be held constant as width increases, a lower and lower number of transverse river is expected to flow through the mountain front.

Castelltort & Simpson (2006), on the other hand, proposed that the drainage acquires its organisation by downstream coalescence of rivers on yet undissected surface outside the mountain range, and that such drainage is progressively incorporated into the uplifted topography as range widening occurs.

A discussion of the reasons for the regularity of valley spacing is beyond the scope of this paper, and will be object of a future paper in preparation. Here, we describe an experiment that simulates mountain front development, and investigate how this replicates first-order topography and drainage characteristics such as the regularity of valley spacing.

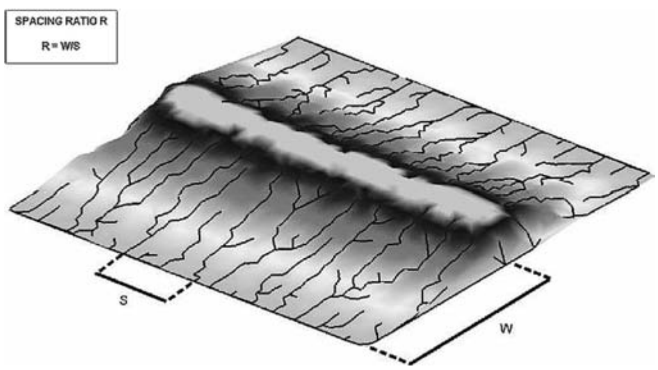

FIG. 3 - Valley spacing ratio R as a function of spacing S and front width W (Hovius 1996).

The geomorphological experiment

We used SIGNUM to simulate geomorphological mountain front evolution in thrust tectonic setting. The thrust module uses in this case the equation obtained from active orogens in Himalaya by Lavè & Avouac (2000) that relates the thrust angle with the horizontal propagation and the surface uplift. The thrust-faults module implemented in SIGNUM simply assumes that structural uplift (u) is proportional to the sine of the local bedding dip angle (θ) and to the horizontal shortening rate (d), as indicated in figure (4). This kind of simulation can provide useful information about thrust-belt development and their influence on landscape features, such as valley spacing and drainage network.

We performed different experiments at different timescales, letting our synthetic landscapes evolve for periods up to 5 My.

The domain is a TIN of 5 x 20 km and the initial condition is a slightly sloped (1°), undissected surface, sampled randomly on 15000 points. The thrust surface has an inclination of 30° and propagates at a horizontal rate of 1 mm/yr simulating mountain widening.

The erosion laws are those related to bedrock river incision and non linear hillslope diffusion with a threshold slope angle of 45° (Roering & *alii*, 1999). At each epoch, the topography is recorded and the valley spacing ratio R at the mountain front is computed.

We are aware that this kind of analysis is based on many simplifying assumptions, such as a constant precipitation regime and erodibility, nevertheless here we are able to show that a simple set of rules representing fundamen-

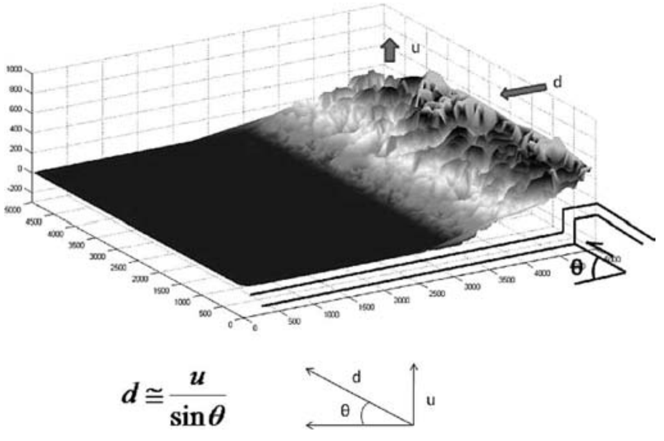

FIG. 4 - Scheme of thrust fault simulation in SIGNUM (see text for parameters).

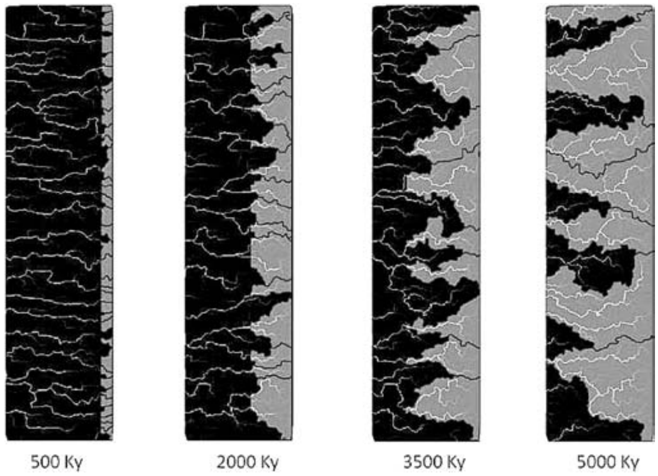

FIG. 6 - Plan view of the simulated drainage network at the mountain front for the same simulation represented in fig. 5, at increasing time steps. Catchments draining at the mountain front from the main drainage divide are also indicated.

tal surface processes can simulate topographic features which exhibit realistic statistical features.

Some topographies resulting from the experiments are shown in fig. 5 and 6. It can be noticed the regularity of the drainage features flowing through the mountain front as the range widens. Next, we try to answer to the following questions: is this regularity proportional to the width of the chain? And what is the value of *R* simulated by the model?

In figure 5 we plot the mean value of *R* and its standard deviation over the width of the simulated mountain front, computed every 500 kyr, for a total span of 5 Ma.

The value of *R* varies between 0.4 and 3, its mean values calculated over the simulated mountain front vary between 1.8 and 2.1. It appears that after an initial increase, the value stabilizes around a value of 2.

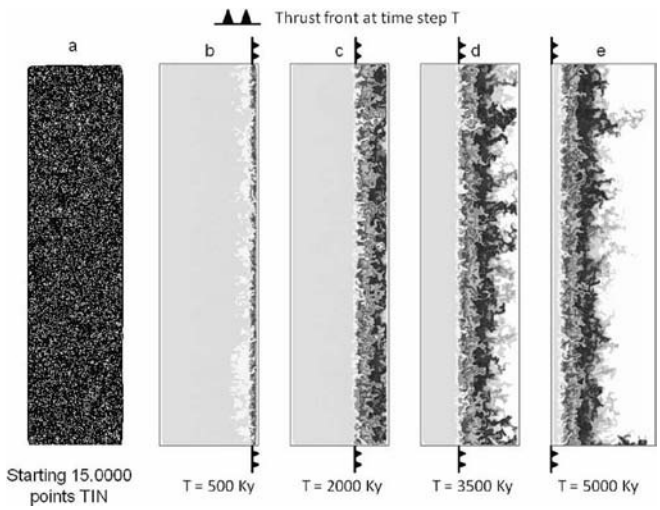

FIG. 5 - Simulation of a widening mountain front. (a) initial TIN of size 5 km x 20 km, sampled randomly over 15000 points; (b)-(c)-(d)-(e); evolution of the initial surface after 500, 2000, 35000 and 5000 ky, respectively. Run parameters are: $k_l = 0.005\ m^2/y$, $k_i = 10^{-5}\ m/y$, m = 0.5 n = 1 (exponents in the equation for bedrock incision), uplift rate = 0.5 mm/y.

The asymptotic value of *R* predicted by the model through the mentioned simple rules to simulate bedrock channel incision, hillslope diffusion and thrust propagation is very similar to that calculated for real mountain chains. It looks like the competition between hillslope diffusion, bedrock channel incision and surface uplift drives the drainage network toward a reorganization during mountain front widening. In particular, we can observe several fluvial captures of small basins by larger ones. So the number of basins that develop up to the main divide decrease while the chain expands, making the spacing between the main outlets increase at the mountain front. The spacing (*S*) and the chain half-width (*W*) increase at such rates that their ratio (*R*) oscillates around a value of 2 in average.

Hovious (1996) explained this regularity in terms of Hack's law, which describes the relationship between stream length and drainage area in a fluvial basin. Although there is no widely accepted explanation for the mechanisms underlying Hack's law, it may be said that the drainage network organizes following an optimal catchment geometry that represents the most probable energy state in linear mountain belts. Our experiments suggest that this regularity is obtained at the early stage of landscape development (fig. 7), in agreement with data from real linear mountain belts where the spacing for very young ranges do not differ from much older ones (Hovius, 1996). Further, our analyses indicate that the simple competition in the uplift-erosion system between hillslope diffusion and channel incision may be the first-order factor that controls drainage development and landscape dissection in real landscapes.

A deeper analysis is still needed to fully elucidate the causes of the regularity of valley spacing, a geomorphological feature described not only at mountain front but also outside orogens or under the sea water and even on other planets like Mars.

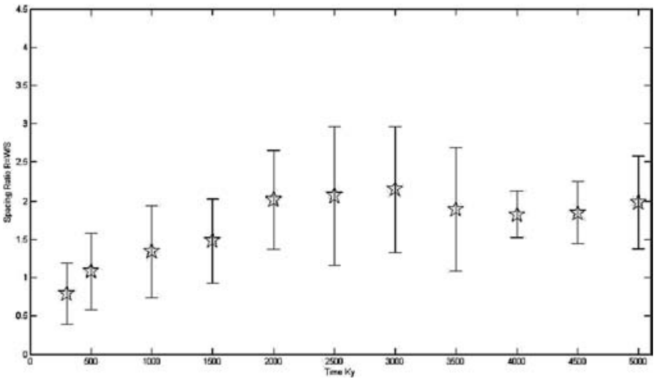

FIG. 7 - Plot of average valley spacing ratio $R = S/W$, calculated along the simulated mountain front in fig. 5 every 500 ky. Error bars have widths equal to the R standard deviation.

CONCLUSIONS

Geomorphology is going toward a period of great development driven mainly by the availability of high resolution topographic and geochronological data and high performance computing environments. One of the pillars of this development is the principle of conservation of mass. Exercising the principle of conservation (Anderson, 2008), either in the field (with natural experiments) or with numerical models, we can explore how factors such as hydrology, vegetation, material properties and humans can affect landform distribution in the landscape and their rate of change. We presented here a simple numerical experiment using SIGNUM, our Matlab, TIN-based landscape evolution model. We show that, by applying simple rules that simulate erosion and transport of selected surface processes such as hillslope degradation, bedrock river incision and surface uplift due e.g. to thrusting, we are able to reproduce quite common features of linear mountain fronts, such as regular valley spacing and drainage diversion and captures.

We also find that our numerical experiments reproduce the characteristic morphometric value of the valley spacing ratio R that many authors found in several linear mountain front in different climatic and tectonic settings. This suggests that the regularity of drainage organization is obtained at the early stage of belt development, in agreement with observation in real landscapes, and that the competition between hillslope diffusion and channel incision can be the first-order factor that controls drainage development and landscape dissection. These encouraging results strongly support the need for new research, new field studies and novel analyses in the field of landscape evolution models.

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