Geogr. Fis. Dinam. Quat.
26 (2003), 29-34, 4 figg., 1 tab.

MARTA DELLA SETA, MAURIZIO DEL MONTE, PAOLA FREDI,
ELVIDIO LUPIA PALMIERI & PAOLA SBARRA (*)

RELATIONS BETWEEN MORPHODYNAMICS AND FRACTAL DIMENSION OF RELIEF IN SOME STUDY AREAS OF ITALY

ABSTRACT: DELLA SETA M., DEL MONTE M., FREDI P., LUPIA PALMIERI E. & SBARRA P., *Relations between morphodynamics and fractal dimension of relief in some study areas of Italy*. (IT ISSN 0391-9838, 2003).

Preliminary results of investigations concerning the application of fractal analysis to the study of the main geomorphic features of different Italian areas show several cause-effect relationships between structure, morphogenesis and fractal properties of relief. In particular, the fractal study of contour lines and topographic surfaces clarifies some aspects regarding the recent morphologic evolution of these areas.

KEY WORDS: Fractal analysis, Morphodynamics, Quantitative Geomorphology.

RIASSUNTO: DELLA SETA M., DEL MONTE M., FREDI P., LUPIA PALMIERI E. & SBARRA P., *Rapporti tra morfodinamica e dimensione frattale del rilievo in alcune aree di studio italiane*. (IT ISSN 0391-9838, 2003).

In questa nota vengono esposti i risultati preliminari di una serie di ricerche incentrate sull'analisi frattale di alcune delle principali caratteristiche geomorfologiche di diverse aree italiane. In particolare, l'analisi frattale applicata alle isoipse e alle superfici topografiche ha consentito di chiarire alcuni aspetti riguardanti l'evoluzione morfologica recente di tali aree, mostrando una evidente relazione di causa/effetto tra la struttura geologica, la morfogenesi e le caratteristiche frattali del rilievo.

TERMINI CHIAVE: Analisi frattale, Morfogenesi, Geomorfologia quantitativa.

Fractal analysis has been successfully applied over the last decades to the study of a variety of natural complex phenomena (Mandelbrot, 1977; Turcotte, 1992; Tokunaga, 2000). The development of this analysis has to be correlated to the clear inadequacy of traditional methods, based on the Euclidean geometry, in providing satisfying genetic and evolutionary models.

In fact, numerous landscape features, such as drainage patterns, coastlines and some karstic morphosculptures, show an intrinsic geometric irregularity significantly quantified by the *fractal dimension* (D_f).

Several Authors discussed the fractal nature of drainage basins and networks (Tokunaga, 1984, 2000; Marani & *alii*, 1991; Masek & Turcotte, 1993; Beer & Borges, 1993; Nikora, 1994; Del Monte & *alii*, 1999, and references therein; Cheng & *alii*, 2001), as well as of topographic surfaces (Sung & *alii*, 1998, and references therein).

In this note preliminary results of long lasting investigations are presented; these investigations concern the fractal analysis of the main geomorphic features of some Italian areas. Particularly, the fractal study of contour lines and topographic surfaces has concurred to clarify some aspects of the recent morphologic evolution of these areas, supporting and integrating results issued by previous analyses performed on drainage networks (Del Monte & *alii*, 1997).

The contour lines of each examined area have been digitised at an operating scale of 1:25,000, and relative DEMs have been generated. The following methods have been used to calculate D_f values: *box-counting* (Goodchild, 1982), *compass-walk* (Carr, 1995), deriving from the *line-divider* one (Mandelbrot, 1967), and *variogram method* (Mark & Aronson, 1984).

Analysing the fractal dimension of contour lines the assumption has been made that contour lines may show linear fractal behaviour as well coastlines (Mandelbrot, 1967).

Obtained results seem to be encouraging: the values of D_f parameter calculated with different methods are clearly related to the kind of processes that shape the relief and to the time interval in which the various exogenous agents have been active. From the geomorphologic point of view this represents one of the most important outcomes.

As an example, the fractal analysis of the contour lines of the Salina Island (Eolie Archipelago, Sicily), morpho-

---

(*) *Dipartimento di Scienze della Terra, Università degli Studi di Roma «La Sapienza».*

This research was funded by the Ministry for Instruction, University and Research (MIUR), Director of Research Prof. E. Lupia Palmieri.

logically characterised by the presence of two main volcanic cones (figs. 1, 2), has evidenced the relation between D_f values and the topographic effects produced by different exogenous processes active in the past as well as at present times.

The analysis of contour lines between 0 and 900 m a.s.l., sampled at an interval of 100 m, clearly shows that the geometry of these lines varies with the elevation, according to the predominant exogenous process. The coastline shows the less indented geometry, outlined by a D_f value close to 1 (tab. 1). This is a consequence of marine erosion that induces the progressive simplification of the shoreline geometry, in the lack of significant opponent processes. On the contrary D_f values progressively increase with sinuosity and linear irregularities of contour lines where the landmass is dissected by fluvial incision. More precisely, in the altimetric range between 100 and 900 m a.s.l., where erosion by channeled surface waters is predominant, the geometry of contour lines becomes more complex. (It is noteworthy that D_f values at elevations of 700-900 m a.s.l. could be influenced by the small length of contour lines. Thus D_f values have been considered significant when $L > 7$ km and the probability that R^2 values are accidental is $< 0.1\%$.)

In areas where argillaceous lithologies crop out fluvial deepening can be easily accompanied by the effective badlands erosion and, as a result, contour lines appear markedly indented. This is the case of Torrente Bretta drainage basin (Marche, Adriatic slope of Central Italy), where badlands are concentrated in a range of altitudes between 250 and 400 m a.s.l.: the highest values of D_f have been calculated for the contour lines right within this altitude range (tab. 1).

A further interesting result of the researches so far carried out is the «fractal zonation» of the study areas. In the case of Salina Island contour lines of the eastern and older

TABLE 1 - Fractal dimension of contour lines for study areas, calculated using two different methods (*box-counting* and *compass-walk*). Along with R^2 values, the degree of freedom (Deg. freed.) used for the linear regression is indicated

SALINA ISLAND

Box counting	**0**	**100**	**200**	**300**		**400**		**500**		**600**		**700**		**800**		**900**
				W	E	W	E	W	E	W	E	W	E	W	E	
D_f	1.06	1.10	1.14	1.14	1.09	1.06	1.23	1.07	1.14	1.06	1.13	0.66	0.93	0.62	1.11	1.00
R^2	0.980	0.999	0.995	0.996	0.999	0.999	0.994	0.999	0.993	0.996	0.996	0.916	0.998	0.962	0.981	0.950
Deg. freed.	7	5	5	5	5	5	5	3	5	3	5	5	3	5	4	4
Compass walk	**0**	**100**	**200**	**300**		**400**		**500**		**600**		**700**		**800**		**900**
				W	E	W	E	W	E	W	E	W	E	W	E	
D_f	1.04	1.10	1.15	1.16	1.18	1.13	1.14	1.10	1.18	1.07	1.18	1.27	1.13	1.35	1.28	1.44
R^2	0.999	0.998	0.998	0.999	0.999	0.996	0.998	0.997	0.998	0.999	0.995	0.988	0.997	0.996	0.982	0.98
Deg. freed.	3	3	3	3	3	3	3	3	3	3	3	3	3	1	3	3

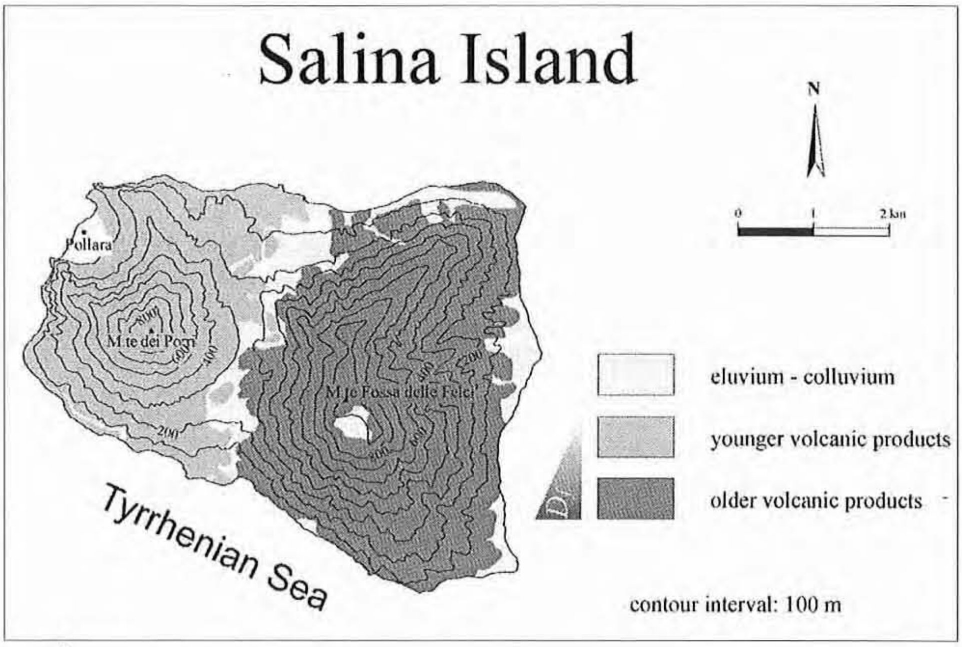
TORRENTE BRETTA BASIN

Box counting	**100**	**150**	**200**	**250**	**300**	**350**	**400**	**450**	**500**	**550**
D_f	1.06	1.07	1.14	1.20	1.20	1.10	1.09	1.04	1.02	1.02
R^2	0.994	0.996	0.999	0.997	0.998	0.999	0.999	0.995	0.998	0.999
Deg. freed.	2	5	5	4	4	5	3	5	4	4
Compass walk	**100**	**150**	**200**	**250**	**300**	**350**	**400**	**450**	**500**	**550**
D_f	1.08	1.10	1.22	1.28	1.22	1.27	1.26	1.21	1.18	1.21
R^2	0.988	0.993	0.996	0.998	0.976	0.997	0.996	0.997	0.997	0.997
Deg. freed.	2	3	3	3	2	3	3	3	3	3

FOSSO DELLA MOLA BASIN

Box counting	**25**	**50**	**75**	**100**	**125**	**150**	**175**	**200**	**225**	**250**
D_f	1.11	1.03	1.11	1.17	1.14	1.11	1.09	1.17	1.31	1.15
R^2	0.995	0.998	0.996	0.995	0.994	0.999	0.999	0.998	0.992	0.997
Deg. freed.	5	5	5	5	5	5	5	5	5	5
Compass walk	**25**	**50**	**75**	**100**	**125**	**150**	**175**	**200**	**225**	**250**
D_f	1.12	1.14	1.12	1.15	1.31	1.21	1.18	1.17	1.17	1.18
R^2	0.997	0.992	0.999	0.991	0.989	0.985	0.989	0.993	0.997	0.995
Deg. freed.	3	3	3	3	3	3	3	3	3	3

FIG. 1 - Schematic map of Salina Island showing analyzed contour lines.



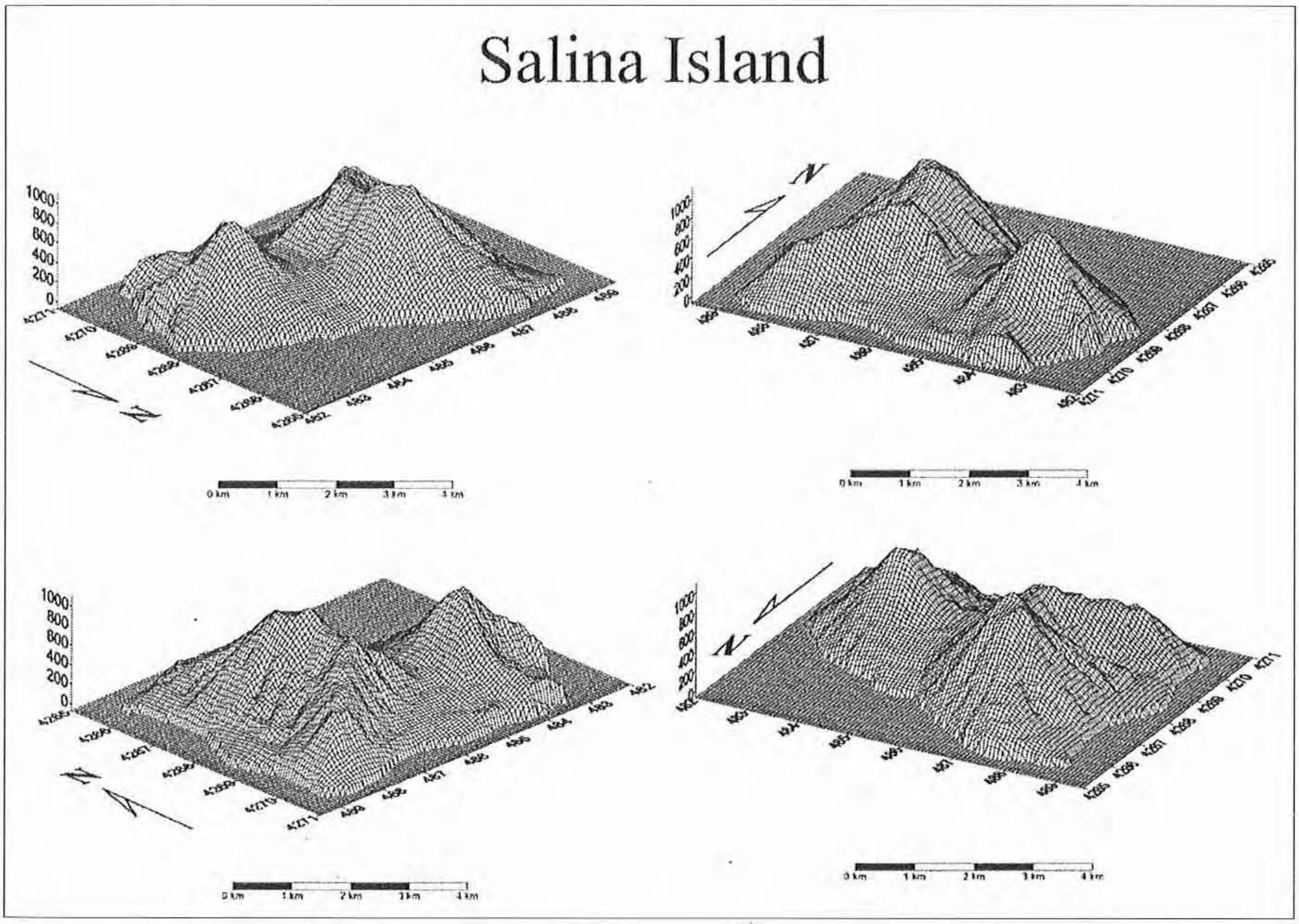


FIG. 2 - 3D views of the Salina Island DEM.

portion of the volcano clearly show D_f values higher than those relative to the western and younger one (tab. 1). Taking into account the similar erodibility of outcropping deposits (Barca & Ventura, 1991), this difference can be attributed to the longer time interval in which the exogenous morphogenetic agents have been operating in the eastern portion of the volcano.

To better define the relationships between time available for denudational processes and fractal properties of relief, the area of the Somma-Vesuvio Volcanic Complex (Campania, Southern Italy) has been examined. In this case fractal dimension of contour lines has been calculated within unit areas (1 km^2), using the *box-counting* method. Obtained results outline that D_f values are directly proportional to the age of outcropping deposits. In particular, D_f values higher than 1.1 have been registered where heterogeneous deposits with an age of 20,000 BP - 79 AD crop out. On the contrary, the portions of the volcano covered by younger lava flows (emplaced between 1631 and 1944) have $D_f < 1.1$ (fig. 3; Davoli & *alii*, 2001).

The fractal analysis applied to the drainage basin of Fosso della Mola (Lazio, Tyrrhenian slope of Central Italy) afforded interesting results too, supporting the hypothesis that fractal dimension of contour lines is influenced both by the type of process and the time elapsed since it began to operate. In this basin D_f values obtained using methods proposed by Goodchild (1982) and Carr (1995), are quite similar for all different contour lines, generally remaining slightly above 1.1. The homogeneity of these values can be related to the existence of a single geomorphic process sensibly influencing the recent morphogenesis of the area, that is fluvial erosion. Moreover, a main morphostructural style dominates this landscape: a tabular relief slightly dipping towards the Tyrrhenian Sea and dissected by a series of valleys of different sizes. This relief is constituted by the pyroclastic products coming from the Sabatini Volcanic Complex; the summit structural surface corresponds to the top of the Bracciano pyroclastic flow erupted about 0.177-0.09 My ago (De Rita & *alii*, 1993) and represents the common reference plain for the beginning of fluvial deepening.

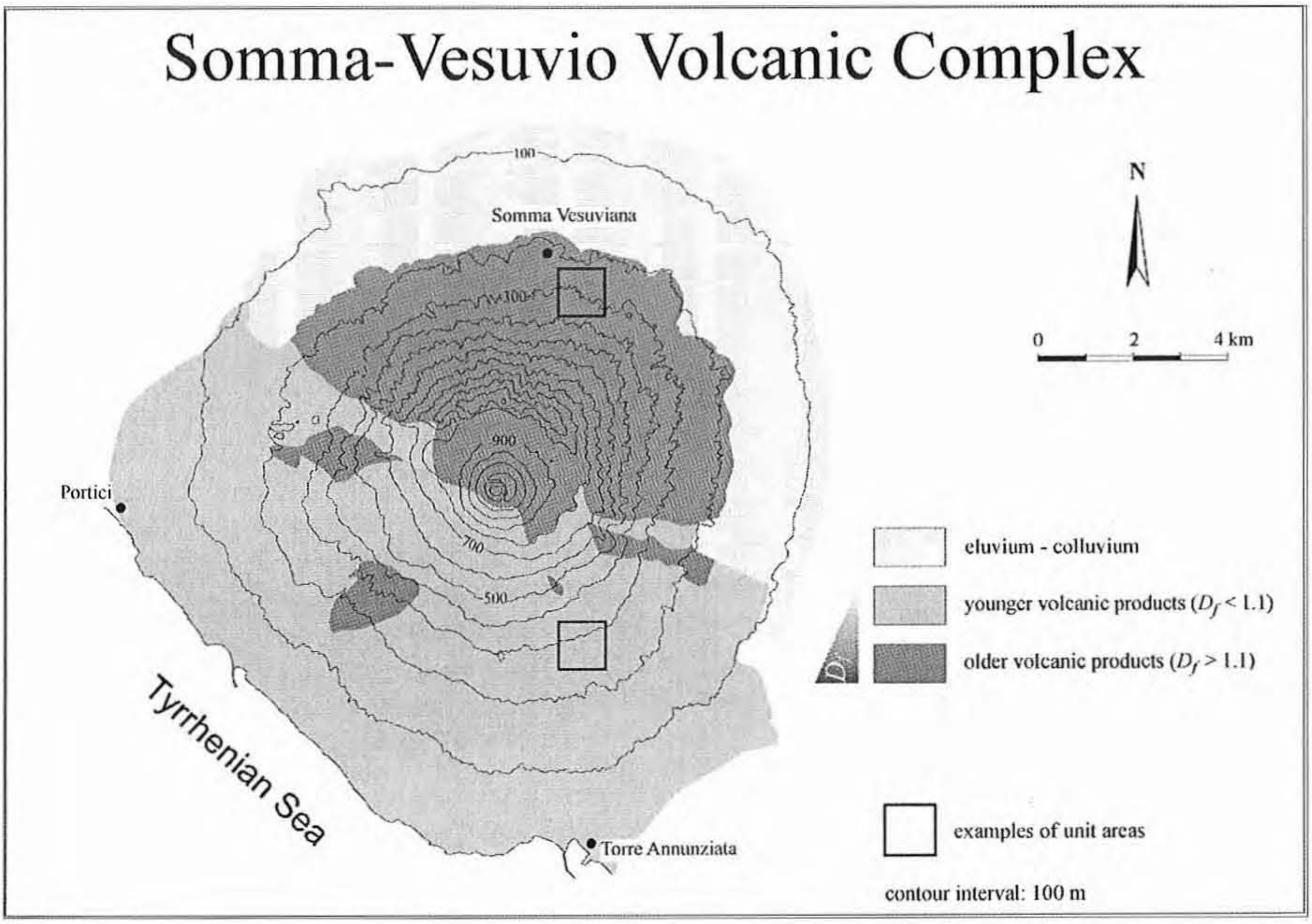


FIG. 3 - Schematic map of the Somma-Vesuvio Volcanic Complex. Fractal dimension has been calculated within unit areas (1 km^2).

On the whole fluvial deepening has cut the relief surface of this area uniformly in space and for the same time span, giving rise to a self-similar morphology for a wide portion of the basin.

Confirming the cause-effect relationship between morphogenesis, structure and fractal properties of relief, unusually high values of D_f have been calculated, within the mentioned basin, for those contour lines representing the rhyolitic domes. These domes, of older age, emerge from the volcanic *plateau* and are quite uniformly spaced within the drainage basin. Therefore they interrupt the morphological continuity of fluvial landscape and induce a further geometric complication of contour lines.

The fractal analysis of topographic surfaces has been tentatively performed. Its outcomes were in agreement with the results of contour line analysis; moreover they allowed defining quantitatively some differences in the morphodynamic conditions of the study areas.

Where denudation by surface running waters is not particularly effective, the values of D_f, calculated through the *variogram method* (Mark & Aronson, 1984), are lower that those obtained in areas affected by more intense erosional processes performed by the same exogenous agent. In fact, in the case of Salina Island the value of D_f is 2.32, whereas in the Torrente Bretta basin it attains 2.54 (fig. 4).

The experimental variogram (fig. 4) computed for the Fosso della Mola basin has a characteristic shape fitting with a *hole-effect* model (Cressie, 1991), thus making it impossible to determine the D_f parameter using the graph. According to Cressie (1991) this typical shape of the variogram suggests that processes generating morphological features with spatial periodicity influence the spatial variation of altitudes within the topographic surfaces of the study area. Analogously to the results obtained for contour lines in the same area, this topographic distribution can be explained by means of the presence of rhyolitic domes previously mentioned. These domes negatively interfere with the scale-invariant plano-altimetric model that the present topographic surface tends to, under the effects of erosion by surface running water.

Summarizing, D_f values of contour lines and topographic surfaces so far considered seem well related to the type and the effectiveness of the geomorphic processes that shaped the relief, as well as to the present morphoevolutive trends. Conditions being equal, fractal analysis of contour lines gives the opportunity of estimating the relative time spent by exogenous agents to exert their modelling activity in different areas.

Actually, results so far obtained encourage investigating more deeply the relations between D_f values and morphoevolutive characters of drainage basins.

Future researches will be focused on the geomorphic significance of D_f values with the aim of developing a quantitative integrated method suitable for the fractal characterization of single features of relief as well as topographic surface as a whole. To this end the *variogram method* seems to be particularly appropriate. In this context, the planned fractal analysis of the Arcinazzo karstic *plateau* (Lazio, Central Italy), where also the allometry of dolinas is going to be studied, seems particularly promising.

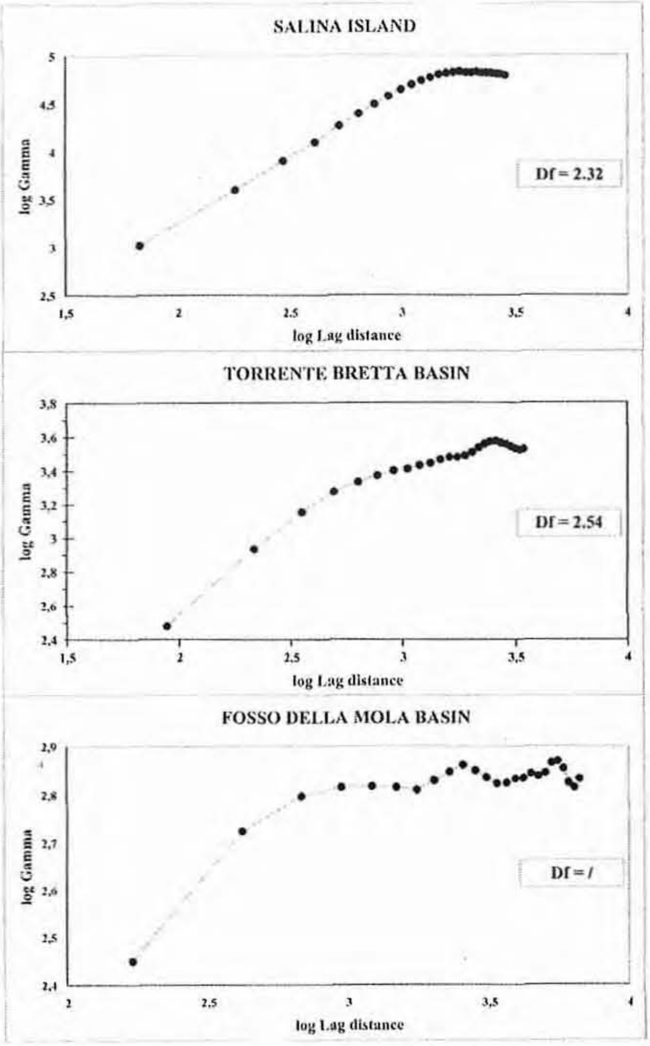


FIG. 4 - Variogram plots of topographic surfaces computed for study areas.

REFERENCES

Barca D. & Ventura G. (1991) - *Evoluzione vulcano-tettonica dell'Isola di Salina (Arcipelago delle Eolie).* Mem. Soc. Geol. It., 47, 401-415.

Beer T. & Borges M. (1993) - *Horton's laws and the fractal nature of stream.* Water Resour. Res., 29, 1475-1487.

Carr J.R. (1995) - *Numerical Analysis for the Geological Sciences.* Prentice Hall, 592 pp.

Cheng Q., Russell H., Sharpe D., Kenny F. & Qin P. (2001) - *GIS-based statistical and fractal/multifractal analyses of surface stream patterns in the Oak Ridges Moraine.* Computers & Geosciences, 27, 513-526.

Cressie N. (1991) - *Statistics for Spatial Data.* Wiley and Sons, New York, 900 pp.

Davoli L., Del Monte M, Fredi P., Russo F. & Troccoli A. (2001) - *Geomorphological mapping and flood hazards in the Somma-Vesuvio volcanic area (Campania, Italy).* Trans., Japan. Geomorph. Union, 22/4, C/49.

Del Monte M., Fredi P., Lupia Palmieri E. & Salvini F. (1997) - *Fractal characterisation of drainage network geometry.* Geogr. Fis. Dinam. Quat., Suppl. III - Tomo I, 144-145.

Del Monte M., Fredi P., Lupia Palmieri E. & Salvini F. (1999) - *Fractal analysis to define the drainage network geometry.* Boll. Soc. Geol. It., 118, 167-177.

De rita D., Funiciello R., Corda L., Sposato A. & Rossi U. (1993) - *Volcanic Units.* In: M. Di Filippo (ed.), *Sabatini volcanic complex.* Quaderni de «La Ricerca Scientifica», C.N.R., Roma, 33-77.

Goodchild M.F. (1982) - *The Fractal Brownian processes as terrain simulation model.* Modelling and Simulation, 13, 1133-37.

La Barbera P. & Rosso R. (1989) - *On the fractal dimension of river networks.* Water Resour. Res., 25/4, 735-741.

Mandelbrot B.B. (1967) - *How long is the coastline of Britain? Statistical self-similarity and fractal dimension.* Science, 56, 636-638.

Mandelbrot B.B. (1977) - *Fractals: Form, Chance, Dimension.* Freeman, San Francisco, California, CA., p. 365.

Marani A., Rigon R. & Rinaldo A. (1991) - *A note on fractal channel networks.* Water Resour. Res., 27, 12, 3041-3049.

Mark D.M. & Aronson P.B. (1984) - *Scale dependent fractal dimension of topographic surfaces: an empirical investigation with applications in geomorphology and computer mapping.* Mathematical Geol. 16, 671-683.

Masek J.G. & Turcotte D.L. (1993) - *A diffusion-limited aggregation model for the evolution of drainage networks.* Earth and Planetary Sci. Lett., 119, 379-386.

Nikora V.I. (1994) - *On self-similarity and self-affinity of drainage basins.* Water Resour. Res., 30, 133-137.

Sung Q.C., Chen Y.C. & Chao P.C. (1998) - *Spatial variation of fractal parameters and its geological implication.* TAO, 9/4, 655-672.

Tokunaga E. (1984) - *Ordering of divide segments and law of divide segments numbers.* Trans. Japan. Geomorph. Union, 19, 77-90.

Tokunaga E. (2000) - *Dimension of a channel network and space-filling properties of its basin.* Trans., Japan. Geomorph. Union, 21/4, 431-449.

Turcotte D. (1992) - *Fractals and chaos in geology and geophysics.* Cambridge University Press, 221 pp.

(Ms received 1 April 2003; accepted 1 May 2003)