Geogr. Fis. Dinam. Quat.
20 (1997), 309-314

ALVARO MAZZA (*)

THE KINEMATIC WAVE THEORY: A PRIORITY OF THE ITALIAN GLACIOLOGY (DE MARCHI, 1895)

ABSTRACT: MAZZA A., *The kinematic wave theory: a priority of the Italian Glaciology (De Marchi, 1895).* (IT ISSN 0391-9838, 1997).

The Italian priority of the kinematic wave theory, developed by Luigi De Marchi as early as 1895 and 1911, is duly recognized. The theory interprets the relation existing between climatic factors and fluctuations of glacier terminus. The theory had been developed disregarding the glacier mechanics, which, at that time, was scarcerly understood.

After the developments of the theoretical glaciology, starting from 1948, carried out by English physicists and metallurgists, the kinematic wave theory has been rediscovered by English researchers and used to investigate the flood waves along rivers and the road traffic flow perturbations.

In an improved physical and technical frame, English and American physicists exploited the new kinematic wave theory in glaciology and, since then, the theory is considered fundamental in the physical and matematical investigation of the glacier fluctuations, with special regard to the evaluation of the response time of glaciers to climate oscillations. It is exactly the evolution of glaciology that, leaving unaltered De Marchi's concepts, confirms the present validity of the kinematic wave theory.

KEY WORDS: Kinematic wave, Continuum mechanics, Materials science, Plasticity, Surge, Computer simulation, Glaciology.

RIASSUNTO: MAZZA A., *La teoria delle onde cinematiche: una priorità della Glaciologia italiana (L. De Marchi, 1895).* (IT ISSN 0391-9838, 1997).

Si pone in rilievo la priorità italiana della teoria delle onde cinematiche formulata da Luigi De Marchi nel 1895 e nel 1911. La teoria interpreta la relazione esistente tra fattori climatici ed oscillazioni della fronte dei ghiacciai. Essa fu formulata astraendo dalle modalità del moto del ghiacciaio, allora scarsamente note.

Dopo gli sviluppi della glaciologia teorica, a partire dal 1948, ad opera di fisici inglesi, la teoria delle onde cinematiche fu riproposta nel 1955 da ricercatori inglesi ed applicata allo studio delle perturbazioni delle correnti lungo i fiumi ed ai problemi della circolazione stradale.

In un quadro fisico e tecnico più maturo, fisici inglesi e statunitensi riproposero la teoria delle onde cinematiche in campo glaciologico e da allora essa è considerata fondamentale nello studio fisico-matematico dell'evoluzione dei ghiacciai, con particolare riguardo al calcolo del tempo di risposta dei ghiacciai a perturbazioni del clima. È proprio alla luce dell'evoluzione della glaciologia che risaltano le concezioni del De Marchi, sostanzialmente ancora oggi inalterate.

TERMINI CHIAVE: Onde cinematiche, Meccanica dei continui, Scienza dei materiali, Plasticità, Simulazioni al computer, Surge, Glaciologia.

(*) *Via Parini 98 - 20043 Arcore, Milano, Italy.*
The Author wants to thank two anonymous reviewers, the contribution of which has strongly improved both form and content of the present paper.

THE KINEMATIC WAVE THEORY: GENERALITY

Aim of this paper is to underline the Italian priority of the theory of «kinematic waves» travelling down the glaciers, established by Luigi De Marchi, and also to enhance its present validity, even after a century from the date of its birth. The theory was developed independently of the glacier flow mechanics and it holds its validity, even after the introduction of material science, plastic deformation theory and continuum mechanics in glaciology, and, since 1958, it is used to explain the glacier dynamics.

Under «kinematic wave», being triggered by an increase of mass in the upper reach of a glacier, it is understood a bulge on the glacier surface which travels along the glacier at a velocity about 4-5 times higher than its average value. The name «kinematic wave» is due to the fact that no dynamical equations are involved in the derivation of its mathematics. The theory applies only to «unidirectional» glaciers (Hutter, 1983).

The physical reality of this event has been proved towards the end of the last century at the «Mer de Glace» (M. Blanc range; Vallot, 1891-1896 in Lliboutry, 1965: p. 629), and, at the beginning of this century, at the Hintereisferner (Austrian Alps; in Lliboutry, 1965: p. 623); more recently a kinematic wave has photographically surveyed at the Nisqually Glacier, Mount Rainer, WA, USA (Veatch, 1969). It is however to be stressed that this event is quite rare in spectacular form [strong increase in velocity in the glacier section interested by the bulge (thickness increase)].

The propagation of a kinematic wave can cause the glacier terminus to advance, if the Summer thermal regimen is close to a steady state. However, in a recent paper on the secular glacier changes in Northern Sweden (Raper & *alii*, 1996), it is stated that in glacier evolution the accumulation factor is more effective than the temperature-one.

THE FIRST DE MARCHI'S PAPER (1895): «LE VARIAZIONI PERIODICHE DEI GHIACCIAI» (THE PERIODIC GLACIER FLUCTUATIONS)

Luigi De Marchi introduces his theory confuting the previous opinions on the causes and features of the periodic glacier fluctuations, proposed by some known glaciologists of his time (Forel, Richter, and Hess). He states that «each continuous matter flux must obey to the general low of continuity», described by a continuity equation:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} + x = 0 \qquad (1)$$

where:
A = area of the glacier section at the point x, as calculated from the point x = 0, along the longitudinal glacier axis;
Q = ice volume; and
α = ablation (melting).

The application of the continuity laws in glaciology, still basic after a century, is the first witness of the correct base of the theory. Hutter (1983: p. 3) writes: «Basic to ice mechanics - be it the theory of glacier flow, the response of floating ice plates to external loading, and the ice drifting, or the very practical question of ice forces on structures - are the fundamental laws of continuum physics».

Starting to outline his theory, the Author makes the assumption that there is a periodic variation of climate, which causes a corresponding periodic variation of Q (ice mass) and α (ice loss, melting); further, in the assumption that such variations may be expressed by a simple harmonic function, he derives the following equations:

$$\frac{\partial Q_0}{\partial x} = - x_0 \qquad (2)$$

$$\frac{\partial Q_1}{\partial x} = - x_1 \cos(\varepsilon_1 - \varepsilon) \qquad (3)$$

$$Q_1 \frac{\partial \varepsilon_1}{\partial x} - A_1 \frac{2\pi}{T} = x_1 \sin(\varepsilon_1 - \varepsilon) \qquad (4)$$

where:
ε = phase of α,
ε_1 = phase of Q; and
$\pi = \varepsilon_1 - \varepsilon$ is the phase shift between Q (ice volume) and α (ice melting).

Considering the only fundamental term of the trigonometric series expansion, the foregoing equations show, according to De Marchi, some basic laws which govern the flux wave propagation and the related bulge thickness.

Equation (2) says that «in a steady-state glacier the flux diminishes along the glacier for the ice quantity subtracted by melting».

Equation (3) shows that «the amplitude of the ice wave varies together with the descent of the same ice wave, increasing if $\cos(\varepsilon_1 - \varepsilon)$ is negative, and decreasing, if it is positive. This variation depends, necessarily, on the existance of a period not only in snow accumulation but also in ice ablation».

As seen above, putting $\varepsilon_1 - \varepsilon = \pi$ (phase shift between Q and α), De Marchi states: «When the ice wave travels down, ε_1 increases, as it can be derived from eq. (4), whereas ε may be considered constant, as the thermal period takes place about simultaneously along the whole glacier length».

Going on in its reasoning, De Marchi writes: «If in a steady-state water current, we pour a given water quantity, this produces a wave travelling along the current; it will reach the end of the current conserving its initial mass, independently of the velocity, even if, at a given point of the current, there is a suction pump which subtracts a constant water vein. It (the mass) can increase only if, at the instant at which the wave flows at a given point, the suction pump, reducing its capacity, will suck a smaller water quantity; or the mass can decrease if the pump, increasing its suction capacity, will suck more water».

The above considerations are the first section of De Marchi's reasoning, «as the melting operates in each point of a glacier as the above pump, and only a reduction or increase of its capacity during the travel of the kineamtic wave, will cause its amplitude to vary». This reasoning depends on the theory of Bruckner's climate periods, which, at that time, were considered as physically existing but today we know they are not, as the data collected later confirm that the fluctuations of climate do not take place at fixed intervals and constant amplitudes.

In De Marchi's paper some new ideas are introduced, namely the «fluidity» of ice (depending on the seasonal temperature): this shows that De Marchi could understand what would be later demonstrated by mechanical testing (compression, bending, impact strength) of ice: keeping constant all other parameters (grain size, kind of stress, stress gradient, strain rate, etc.) involved, *the response of ice to a stress depends on temperature.*

De Marchi defines the velocity of the wave and its propagation depending on the phase shift between the climatic oscillation and the glacier response. Even if a «glacier mechanics» was missing, he gets to the conclusion, still valid, that *the velocity of the kinematic wave is higher than the velocity of the glacier*; for the first, he gives the following relation:

$$u = \frac{\pi}{T} \cdot \frac{p_1}{h_1} \cdot \frac{S}{b} \qquad (5)$$

where
u = velocity of the kinematic wave;
π = phase shift ($\varepsilon_1 - \varepsilon$) between Q (ice volume) and α (ice melting);
p_1 = increase in precipitation;
h_1 = thickness increase of the glacier;
S = accumulation area;
b = glacier cross section at the outlet from the accumulation area.

De Marchi gives the time τ which the kinematic wave would take to travel along the whole glacier of length l, if its velocity would remain constant at its starting value:

$$\tau = \frac{l}{x} = \frac{1}{\tau} \, Tl \, \frac{h_1}{p_1} \cdot \frac{b}{S} \qquad (6)$$

(with the symbols already specified); the relation «can be read - writes De Marchi - saying that the wave travel time needed is proportional to the period time and to the length and *sensitivity* of the glacier, and inversely proportional to its *promptness*».

Under *sensitivity* the A. means the ratio between h1, i.e. the height of the kinematic wave (bulge on the glacier surface), and p1, the increase in precipitation; under *promptness*, De Marchi means the ratio between S, area of the accumulation reach, and b, cross-section at the exhit from the accumulation reach.

Today the response time quoted by De Marchi is considered too short and the idea of periodical climate fluctuations is no longer valid; but this does not impair the three following fundamental derivations of the first theory of kinematic waves:

1) the glacier must be treated as a continuum and, hence, its evolution must be seen in the frame of the theory which, many decades later, would be called «glacier mechanicas», which is but an extension of the «continuum mechanics»;

2) the kinematic wave travels at a velocity higher than the average value of the glacier;

3) there is a phase shift between the excess in feed (snow fall) and the terminus expansion; today, this is called «response time of a glacier».

DE MARCHI'S SECOND PAPER: «LA PROPAGATION DES ONDES DANS LES GLACIERS» (THE KINEMATICvWAVES TRAVELLING ALONG THE GLACIERS; 1911)

With reference to a paper of Tarr (in De Marchi, 1911), which describes the sudden advance of an Alaskan glacier in the Yakutat Bay, attributed to an earthquake, De Marchi revisites his theory of 1895, giving an up to date version of it which, probably but not expressely, takes into consideration the paper of Finsterwalder (1907), which also concerns the same problem of the kinematic waves, on the base of measurements carried out on the Hintereisferner (Austrian Alps). The theory is of course still based on a continuity equation (say conservation of mass, see eq. 1) and disregards as the former one, the glaciers mechanics «... indépendamment de toute théorie sur la nature et la cause du mouvement du glacier ...» (non depending on any theory on nature and cause of the glacier motion). De Marchi (and S. Finsterwalder too) is conscious that, at his time, the mechanics of mass transfer from the accumulation reach to the ablation-one, was poorly understood.

The three quantities already mentioned, are considered periodic and, hence, expandable in a Fourier series; in this way De Marchi assumes that dQ/dx and α (flux and ablation) correspond to the sum of single waves, each representing a climatic period which is supposed by the A. having the same time span, and such that, at each oscillation of Q corresponds an oscillation of A having the same period. A_0, Q_0 and α_0 are the regular values; A_n, Q_n and α_n are the n^{th} wave amplitudes, and ε_n, ε'_n, and ε''_n are the corresponding phases; all these quantities are function of x (longitudinal axis of the glacier).

De Marchi explains the conditions which satisfy the equation (5) in which the series expansions are introduced. The flux variations of Q, says De Marchi, cannot be separated from the variations of A; nevertheless they may be considered sinchronous, as first approximation. De Marchi's equations are the following:

$$\frac{\partial Q_o}{\partial x} = -\alpha_o \qquad (7)$$

$$\frac{\partial Q_n}{\partial x} = -\alpha_n \cos\left(\varepsilon''_n - \varepsilon_n\right) \qquad (8)$$

$$Q_n \frac{\partial \varepsilon_n}{\partial x} - A_n \frac{2\pi}{T} = -\alpha_n \sin\left(\varepsilon''_n - \varepsilon\right) \qquad (9)$$

Equation (7) shows the evidence that, for a glacier in regular regimen, the flux decreases along the same glacier, due to ice melting.

Equation (8) expresses the law of the wave amplitude variation: the amplitude decreses if the melting period is phase-delayed or anticipated, with reference to the flux period for a quantity less than

$$T/4, \text{ i.e. } (\varepsilon'_n - \varepsilon_n < 90°) \qquad (10)$$

and increases in the opposite case; and it remains unchanged if the phase shift if 90° or if melting is constant ($\alpha_0 = 0$).

Eventually from equation (9) De Marchi derives the velocity of the wave propagation.

This simplification is considered valid by the A. either in case in which the climatic period is very short and the ice melting is constant; or in case in which the melting wave is in phase with that of the flux ($\varepsilon''_n = \varepsilon_n$) as it happens in seasonal periods in which the velocity is higher (Summer) and lower (Winter; we know today that the ice mechanical strength, and, hence, its deformability and, eventually, the glacier velocity, depend on temperature). If we choose the origin of x in way such as, for t = 0, ε' = 0, we obtain C = 0, then, ε'_n (perturbed phase of the flux) is given by:

$$\varepsilon'_n = \frac{2\pi}{T}\left[\frac{A_n}{Q_n}\right] x \qquad (11)$$

$[A_n/Q_n]$ being an average value of the ratio area/flux in the interval x. The n^{th} term of the Q expansion will be given by:

$$Q_n \cos \frac{2\pi}{T}\left\{ t - \left[\frac{A_n}{Q_n}\right] x \right\} \qquad (12)$$

which represents a wave travelling along the x axis at a velocity (Q_n/A_n). This velocity has a meaning and a value completely different from the average value Q_0/A_0 of the flux. To give an approximate value of the wave propagation velocity, De Marchi assumes that the positive half-wave at the origin of motion (t = 0), on a long-section of the glacier L/2 (L = wave length), with x = 0 as average axis, includes the volume q of snow accumulated in amount exceeding the regular value during the half-period T/2.

If b is the glacier width and h_0 is the original height of the wave (bulge), we will have:

$$A_n = b.h_n \qquad (13)$$

The starting velocity of the wave propagation will be:

$$u_o = \frac{\pi}{T}\frac{q}{bh_o} \qquad (14)$$

Be l the length of a glacier from the origin of the wave; let assume that the propagation velocity is uniform; the time τ needed by the wave bulge to reach the glacier terminus will result:

$$\tau = \frac{l}{u_o} = \frac{T}{\pi}\frac{s \cdot h_o}{q} \qquad (15)$$

From eq. (15) De Marchi derives that the time needed by the wave to get to the glacier terminus, causing the increase in thickness and the terminus expansion, is so much shorter when:

1) the shorter is the period T;
2) the bigger is the snow volume accumulated at the origin of the kinematic wave;
3) the smaller is the glacier size; and
4) the «fluider» (today we would say «more ductile») is the glacier ice.

The height h_0 of the wave bulge depend, in fact, on is «fluidity» (this is the word used by De Marchi; later on, one would speak about the ice viscosity depending on temperature, grain size and stress conditions, as already mentioned), as a given ice volume q should transform in a wave (bulge) which is so much flatter on the glacier surface, the «fluider» (= less viscous) is its ice.

MATERIALS SCIENCE, CONTINUUM MECHANICS, AND GLACIOLOGY

It has been already stressed that the above De Marchi's theory is still valid, even if established when the present knowledge concerning the properties of ice as material, as well as a «glacier mechanics» were missing. The science of materials, and specially the investigation on elasticity and plasticity, was started in the half of the past century mainly by Tresca (1864; in Lliboutry, 1964), Barré de St.-Vénant (in Lliboutry, 1964), and, at the beginning of this century, mainly by Von Mises (1913; in Lliboutry, 1963), but, as already said, the knowledge gained had not yet been extended to glaciology, which had been mainly developed by Naturalists than by Physicists, except some exceptions (e.g. Tyndall, 1860).

We are to wait until the historical meeting organized by Seligman on April 29, 1948 with the participation of the British (later International) Glaciological Society, the British Rheologists Club and the Institute of Metals, during which Orowan (1949), a metallurgist, outlined the mathematical theory of a perfectly plastic body (Lliboutry, 1964).

The application of the material science to glaciology, proposed by Orowan (1949) and later by British physicists, mainly Nye (1951) and Glen (1955), marked the start of the modern theoretical glaciology, within which a new theory of kinematic waves travelling along the glaciers has been developed. In 1953 the theory of dislocations, lattice defects to which the deformation of policrystalline materials, such as metals and ice too, is due, has been developed mainly by Cottrell (Lliboutry, 1964). The invention of the electronic microscope confirmed the physical existence of the lattice defects, mainly the dislocations. The application in the field of structural glaciology followed in short time.

Now the theoretical frame for the physical and mathematical outline of the conditions in which the creep (very slow deformation under constant load) of ice takes place, was ready and enabled some researchers to review the kinematic wave theory, in the light of the contributions coming to the glaciology from the development of continuum mechanics, numerical analysis and tensor calculus (Synge & Schild, 1949; Nye, 1959).

THE NEW THEORY OF KINEMATIC WAVES TRAVELLING ALONG GENERICvCURRENTS

In 1955 two British researchers, Lighthill and Whitham, (Hutter, 1983; Paterson, 1994) published their investigations on the kinematic waves to study the flood waves on rivers and the traffic flow peraturbations. The «new» theory of the kinematic waves was later extended to glaciology by Nye and Weertman; it is clearly outlined in the glaciological treatises by Lliboutry (1965), Hutter (1983) and Paterson (1994), to which anyone interested in this subject can easily apply; we give up to outline it, considering that this paper is mainly devoted to enlighten the glaciological component of the wide activity of De Marchi (Castiglioni, 1937).

We only recall a new, important feature of the revisited kinematic wave theory: the introduction of a quantity called «diffusion»; this, depending on the geometrical features of a glacier, exerts its effect reducing the height of the wave bulge and increasing the response time of the glacier to the climate fluctuations (Nye, 1965). The mathematics of the new theory is however quite different (Hutter, 1983; Paterson, 1994), as it is based on the mass balance of glaciers. In a newer paper (Van De Wal & Oerlemans, 1995) the problem of propagation of the kinematic waves in glaciers is critically reviewed: according to the Authors, the wave velocity could be 5 - 8 times the average glacier velocity.

THE GLACIER SURGE AS ASYMPTOTIC LIMIT OF THE KINEMATIC WAVE PROPAGATION

De Marchi's idea (De Marchi, 1895 and 1911) has brought a plenty of results; in our opinion, the concept of kinematic wave is applicable, as limit, to the «surge» of glaciers; in fact some glaciers remain quiescent for long time (it does not seem that they surge at fixed intervals) during which there is a regular accumulation: the main factors triggering the «ice flood» are likely to be a strong increase of thickness, with consequent or accompanying increase in temperature at the glacier bed, where the ice reaches the melting point [the dependance of the ice melting point from the load (pressure) has been already outlined by Tyndall (1860) and quantified by Fermi (1937)].

The increased water bulk, sometimes at high pressure, reduces the ice/bedrock friction, and, hence, the shear strength at the glacier bottom, and eventually the reaction to the glacier motion, with consequent increase in glacier sliding velocity. (According to Van De Ween & Oerlemans (1995) an increased surface velocity, due to increase sliding velocity, could be misinterpreted as a kinematic vawe.)

The result is the transition from the creep (flow of ice under constant stress) to a chaotic flow (as the ultimate tensile strength of ice has been overcome; Molnia, 1995); the final result is an increase in glacier velocity by 10 - 20 time the typical average velocities of glaciers, and even more.

An exemplary limit-case, which supports the above statement of a kinematic wave in transition towards surging conditions, took place at the Tapridge Glacier, Yukon Territory, Canada (Clarke & Blake, 1991); this surge-type glacier on June 24, 1980 showed a dramatic front of kinematic wave close to its terminus, which can be understood as a transition condition from the arrival of the kinematic wave to the terminus to the actual surge; temperature measurements showed that the kinematic wave front marked the limit of transition from the cold ice (about –2 °C) to the temperate ice (close to 0 °C, or, better, 273.15 K), hence with an increase of 2 °C of the surface layer (thickness 15 m) of the glacier. Later the event disappeared, without triggering an actual surge.

METHODS APPLICABLE TO THE COMPUTER SIMULATION OF THE KINEMATIC WAVE THEORY

Within the frame of the theory outlined, it is assumed to be of interest to mention some methods of computer simulation, suitable to investigate and check the theory of the kinematic waves.

We just mention the Finite Element Method (Fem) (Brauer, 1992) and the more recent «Lattice Boltzmann Method» (Bahr & Rundle, 1995); both, based on informatic support, represent interesting investigation possibilities to study deformation and thermal conditions of glaciers. Assuming that the Fem is quite popular, we only give some information on the Lattice Boltzmann Method, an outline of which has been recently published (Bahr & Rundle, 1995); this is likely the most interesting software way to simulate the travel of a kinematic wave along a glacier.

In the study of fluid currents as continua of the macroscopic physics, the well-known Nivier-Stokes equations are currently used; they are partial differential equations (Mosetti, 1979) the solution of which, once very difficult or quite impossible by analytical means, can today be tried by the approximate methods of the numerical analysis (e.g. by the finite difference method). It is well known that the ice may be considered, under some respects, as a non-Newtonian highly viscous fluid, that is a fluid in which, at a linear increase of its velocity, only a logarithmic increase in shear strength takes place; to describe the flow of a glacier, the above-nemtioned Navier-Stokes equations can be used. The finite difference and finite element methods, which take profit from new software of automatic generation of grids suitable to simulate specific problems, treat a fluid as a macroscopic continuum. Recent progress in statistical mechanics has opened new possibilities of creating flow models of low-viscosity fluids around complex geometrical shapes.

The method called «Lattice Gas Automaton» (Lga; Bahr & Rundle, 1995) represents a simulation way of the flow conditions at microscopic scale; it has given excellent results in treating very complicated boundary conditions according to simple rules of particle collision, which reproduce in any point the velocities of the fluid current one needs to investigate. The computer coding is less complicated than that needed for the finite difference equations.

Under «automaton» the AA. mean a computational machine (computer) with a definite set of inputs and outputs; it corresponds to the original concept of von Neumann's computer; at each step of the development of the Lga model, a lattice cell is shifted in the direction of the particle velocity; after the collision has taken place, another cell is processed, and so on. The microscopic model was created for mass, momentum and energy conservation. The Lga models simulate a fluid as sets of gaseous particles in collision, which, statistically, lead to a model of fluid flow of Nevier-Stokes type. The transition from the macroscopic to the microscopic size allows a strong progress in modelling the glacier flow to be obtained.

An evolution of the Lga method has brought to the development of the Lattice Boltzmann Method (Bahr & Rundle, 1995); the method considers populations of particles rather the individual particles, as in Lga method. Practically it is as if someone operates simultaneously with a wide set of Lga simulations, determining later the probability that a cell has a given momentum, i.e., that it occupies a particular position in the lattice. The Lattice Boltzmann method is suitable to treat problems of isothermal steady-state conditions or involving *variation of velocity in glaciers*, maintaining unaltered their geometry. This technique may be used to transform the surface distribution of glacier velocity, to obtain information of the conditions existing at the ice-bedrock interface.

The method looks like to be very promising for the simulation of the kinematic wave travel along the glaciers: the actual difficulty lies in surveying the events of kinematic waves; method like the Dtm (Digital Terrain Model, referred to the topographic surface) or, better, the Dem (Digital Elevation Model, taking into account the accidents on the topographic surface) could be used to survey glaciers, at a given time interval, which are subject to kinematic waves.

CONCLUSIONS

The priority of the kinematic wave theory, due to an Italian scientist, geophysicist, geographer and glaciologist (Castiglioni, 1937), receives its confirmation, after more that a centrury from its birth, from the investigation which are steel carried out on it.

It seems significant to underline that the severe Hutter (1983) in his treatise «Theoretical glaciology» takes in consideration the contribution of only three Italian scientists to the theoretical glaciology: de Marchi, the matematician Volterra, to which the first mathematical formulation of a viscoelastic body is due, as well as the idea, very advanced for the time he published it (Volterra, 1912)[1], of heredity of materials structures, rediscovered in the 70's in metallurgical field; and, eventually, Tricomi, another well-known mathematician: hence, the theory of the kinematic waves in glaciers is the only acknowledgement of K. Hutter to the Italian theoretical glaciology!

[1] Volterra was a member of the Italian Glaciological Committee in 1914.

REFERENCES

BAHR D.R. & RUNDLE J.R. (1995) - *Theory of lattice Boltzmann simulations of glacier flow*. Journ. Glaciol., 41 (139), 634-640.

BRAUER J.R. (1992) - *Analisi agli elementi finiti*. Tecniche Nuove, Milano.

CASTIGLIONI B. (1937) - *L'opera scientifica di Luigi De Marchi*. Mem. R. Acc. Sc. Lett. Arti in Padova, 53, 3-56.

CLARKE G.K.C. & BLAKE E.W. (1991) - *Geometric and thermal evolution of a surge-type glacier in its quiescent phase: Tapridge glacier, Yukon territory, Canada, 1969-89*. Journ. Glaciol., 37 (125), 158-169.

DE MARCHI L. (1895) - *Le variazioni periodiche dei ghiacciai*. Rend. Ist. Lombardo Sc. Lett., 18, 25-37.

DE MARCHI L. (1911) - *La propagation des ondes dans les glaciers*. Zeit. Gletscherk., 5, 507-211.

FERMI E. (1937 reprint 1956) - *Thermodynamics*. Dover, New York.

FINSTERWALDER S. (1907) - *Die Theorie der Gletscherschwankungen*. Zeit. Gletscherk., 2, 81-101.

GLEN J.W. (1955) - *The creep of policristalline ice*. Proc. R. Soc. London, 228, 519-538.

HUTTER K. (1983) - *Theoretical glaciology. Material science of ice and the mechanics of glaciers and ice sheets*. Reidel, Doordrecht.

LLIBOUTRY L. (1964, 1965) - *Traité de glaciologie*. 2 voll., Masson, Paris.

MOLNIA B. (1995) - *Cover picture of the Journal of Glaciology*, 41 (139) *concerning the surge of the Bering Glacier, Alaska*.

MOSETTI F. (1979) - *Oceanologia e idrologia*. Utet, Torino.

NYE J.F. (1951) - *The flow of glaciers and ice sheets as a problem in plasticity*. Proc. R. Soc. London, Ser. A, 207, 554-572.

NYE J.F. (1959) - *A method of determining the strain-rate tensor at the surface of a glacier*. Journ. Glaciol., 3 (25), 409-419.

NYE J.F. (1965) - *The frequency response of glaciers*. Journ. Glaciol., 5 (41), 567-587.

OROWAN E. (1949) - *Remarks at the joint meeting of the British Glaciological Society, the British Rheologits Club and Institute of Metals*. Journ. Glaciol., 1, 231-236.

PATERSON W.S.B. (1994) - *The physics of glaciers*. 3ª ed., Pergamon Press, Oxford.

RAPER S.C.B., BRIFFA K.R. & WIGLEY T.M.L. (1996) - *Glacier change in northern Sweden from ad 500: a simple geometric model of Storglaciären*. Journ. Glaciol., 42 (142), 341-351.

SYNGE J.L. & SCHILD A. (1949; repr. 1978) - *Tensor calculus*. Dover, New York.

TYNDALL J. (1860, reprint 1896) - *The glaciers of the Alps*. Longmans, Green. and Co., London.

VAN DE WAL R.S.W. & OERLEMANS J. (1995) - *Response of valley glaciers to climate change and kinematic waves: a study with a numerical ice-flow model*. Journ. Glaciol., 41 (137), 142-152.

VEATCH F.M. (1969) - *Analysis of a 24-year photographic record in Nisqually Glacier, Mount Rainier National Park, Washington, USA*. U.S.G.S. Prof. Paper 631.

VOLTERRA V. (1912) - *Sur les equations integro-differentielles et leur applications*. Acta Math., 35, 295-356.